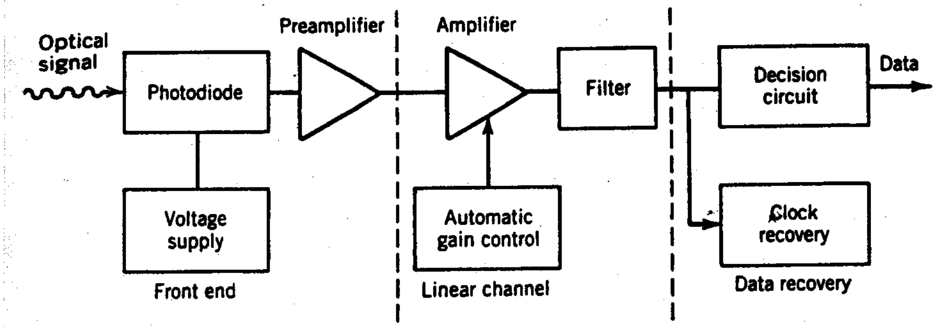
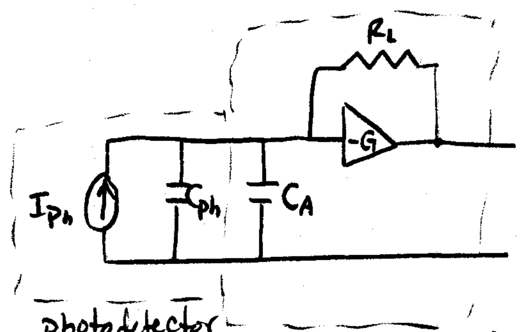
Receivers



The bandwidth is limited by the component with the lowest frequency response.

The most critical component with respect to noise is the Preamplifier.

The photodetector is usually modeled as a current source with capacitance.
The most common Preamplifier is a transimpedance amplifier, which converts current into voltage



transimpedance amplifier (TIA)

The bandwidth is $\Delta f = \dfrac{1}{2\pi R_{in} C_T}$ $\qquad C_T = C_{ph} + C_A$

For high bandwidth C_T and R_L both need to be low

For low noise R_L needs to be high.

For the TIA the input impedance is $\dfrac{R_L}{G}$.

If the gain is high enough the signal to noise ratio is determined by the first amplifier stage (TIA).

Primary noise sources of optical receivers
Shot noise
thermal noise

Shot Noise

Random EHP generation
The noise $i_s(t)$ is a stationary process with Poisson statistics
The spectral density

$$S_{shot}(\omega) = q\,(I_{ph} + I_d) \qquad \text{constant with frequency}$$

The noise is limited by using a low pass filter slightly higher than the data rate.

The noise variance is;

$$\sigma_{shot}^2 = \langle i_s^2(t) \rangle = \int_{-\infty}^{\infty} S_{shot}(f)\, df =$$

$$\sigma_{shot}^2 = 2q(I_{ph} + I_d)\,\Delta f \qquad (\Delta f \approx B)$$

Thermal Noise

Random motion of electrons in a resistor
The load resistor in the front end of the optical receiver causes fluctuations to the generated current

$i_T(t)$ is a stationary Gaussian random process

$$S_{thermal}(\omega) = \frac{2 K_B T}{R_L}$$

Again we limit the noise by using a low pass filter

$$\sigma_{th}^2 = \langle i_T^2(t) \rangle = \int S_{thermal}(f)\, df$$

$$\sigma_{th}^2 = \frac{4 K_B T}{R_L}\,\Delta f$$

The actual amplifier introduces additional noise. This is characterized by F_n : amplifier noise figure.

$$\sigma_{th}^2 = \left(\frac{4 K_B T}{R_L}\right)(F_n)\,\Delta f$$

Since σ_{sh} and σ_{th} are independent random processes

$$\sigma_n^2 = \sigma_{sh}^2 + \sigma_{th}^2$$

The signal to noise ratio is

$$SNR = \frac{\text{average signal power}}{\text{noise power}}$$

resulting in

$$SNR = \frac{(R\,P_{in})^2}{\left[2q\,(R P_{in} + I_d) + \frac{4 k_B T}{R_L} F_n\right] \Delta f}$$

In order to maintain a high SNR the signal is low pass filtered very close to the bit rate $\Delta f = B$.
The SNR scales with bit rate.

$$SNR = SNR(B_0)\,\frac{B}{B_0}$$

For example:

if the $SNR = 100$ at $B = 1\,Mbps$
$SNR = 0.1$ at $B = 1\,Gbps$

So as the bit rate increases the required received power also increases. (A reduction in link length)

The best possible receiver is shot noise limited.

$$SNR = \frac{(R\,P_{in})^2}{2q\,R P_{in}} = \frac{R\,P_{in}}{2qB} = \frac{\eta\,P_{in}}{2hfB}$$

In order to eliminate the frequency dependence it is often related to the number of photons per bit.

$$P_{in} = (\#\text{ of photons})\,\cancel{hf} = \frac{SNR\,(2\cancel{hf})\,B}{\eta}$$

$$\frac{(\#\text{ of photons})}{B} = N_p = SNR\left(\frac{2}{\eta}\right)$$

$$SNR = \frac{\eta\,N_p}{2}$$

For example: An ideal receiver with $SNR = 10$ requires $\frac{20}{\eta}$ photons
$P_{in} = 20$ Photons/bit (independent of wavelength)

Most practical receivers are more thermal noise limited. To characterize the actual receiver performance you can use a term called Noise Equivalent Power (NEP)

Definition: Required incident optical power for $SNR = 1$ per unit bandwidth

$$1 = \frac{(R\ NEP)^2}{\left[2q\,(R\,NEP + I_d) + \frac{4 k_B T}{R_L} F_n\right](1)}$$

$$(R\ NEP)^2 - 2q\ R\ NEP - 2q\,I_d - \frac{4 k_B T}{R_L} F_n = 0$$

$$(NEP)^2 - \left(\frac{2q}{R}\right) NEP - \frac{2q}{R^2}\left(I_d + \frac{2 k_B T}{q R_L} F_n\right) = 0$$

$$NEP = \frac{q}{R}\left[1 + \sqrt{1 + \left(\frac{2}{q}\right)\left(I_d + \frac{2 k_B T}{q R_L} F_n\right)}\ \right]$$

unless cryogenically cooled $\frac{4 K_B T}{q^2 R_L} F_n \gg 1$

$$NEP = \frac{q}{R}\left[1 + \sqrt{\frac{2}{q}\left(I_d + \frac{2K_B T}{q R_L} F_n\right)}\right]$$

$$= \left(\frac{q}{R}\right)\sqrt{\left(\frac{2}{q}\right)\left(I_d + \frac{2 K_B T}{q R_L} F_n\right)}$$

$$R\, NEP = \sqrt{2q\, I_d + \frac{4 K_B T}{R_L} F_n}$$

So the NEP characterizes everything except for the shot noise

$$SNR = \frac{(R\, P_{in})^2}{\left[(R\, NEP)^2 + 2qR\, P_{in}\right]\Delta f}$$

How do APDs help with SNR?

The produce photo current increases $I_{ph} = M R P_{in}$

The thermal noise stays the same.
The shot noise results from the randomness of EHP recombinations. The multiplication factor itself becomes a random variable. The shot noise becomes

$$\sigma_{sh}^2 = 2q M^2 F_A (R P_{in} + I_d)\Delta f$$

F_A is the excess noise factor

$$F_A = k_A M + (1 + k_A)(2 - 1/M)$$

$K_A = \alpha_h/\alpha_e$ or $\frac{\alpha_e}{\alpha_h}$ which ever is smallest

	Si	Ge	InGaAs
k_A	0.02 - 0.05	0.7-1.0	0.5-0.7
M	100 - 500	50-200	10-40
I_d (nA)	0.1-1	50-500	1-5

If the receiver is shot noise limited an APD has worse performance. Most receivers are thermal noise limited so

thermal limited $\quad \frac{SNR_{APD}}{SNR_{pin}} = \frac{(M R P_{in})^2}{\frac{4k_BT}{R_L} F_n \Delta f} \quad \frac{\frac{4k_BT}{R_L} F_n \Delta f}{(R P_{in})^2} = M^2$

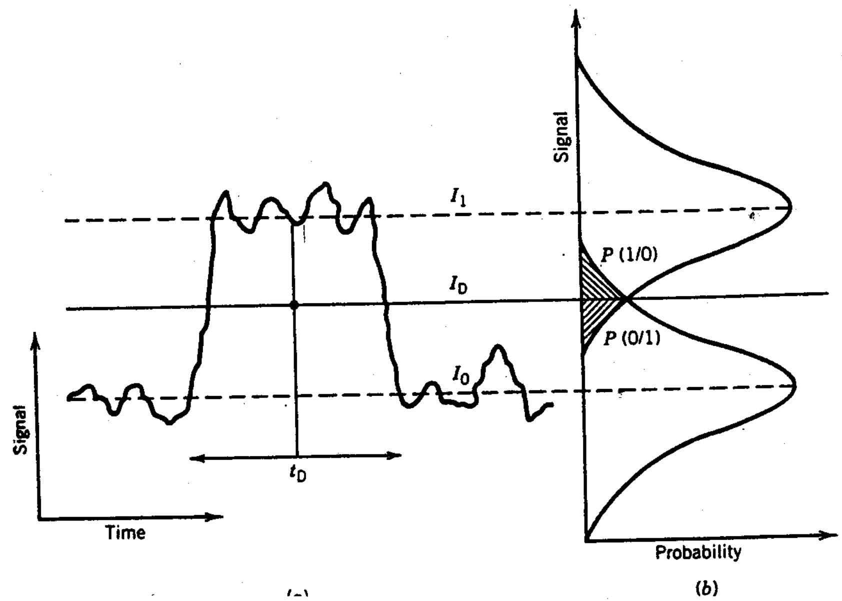
shot noise limited $\quad \frac{SNR_{APD}}{SNR_{pin}} = \frac{(M R P_{in})^2}{2q M^2 F_A R P_{in}} \quad \frac{2q R P_{in}}{(R P_{in})^2} = \frac{1}{F_A}$

Since most receivers are thermal noise limited APDs have superior performance.

Receiver Sensitivity

Bit Error Rate (BER) : Probability of an incorrect bit

$BER = 10^{-9}$ means 1 wrong bit out of 10^9 bits



Two possible errors : choose 1 when a Ø was sent
choose Ø when a 1 was sent

$$BER = p(1)\, P(0/1) + p(0)\, P(1/0)$$

since $p(1) = p(0) = 1/2$

$$BER = \frac{1}{2}\left(P(0/1) + P(1/0) \right)$$

The thermal noise power is independent of whether a "1" or "0" was sent $\sigma_{Th}^2 = \frac{4k_B T}{R_L} \Delta f$

The shot noise is signal dependent and is given by

$\sigma_{sh}^2 = 2q\,(RP_{in} + I_d)\,\Delta F$ Pin

$$\sigma_{sh}^2 = 2q M^2 F_A\,(RP_{in} + I_d)\,\Delta f$$

The resulting noise powers are σ_0, and σ_1

The error probabilities become

$$P(0/1) = \frac{1}{\sigma_1\sqrt{2\pi}} \int_{-\infty}^{I_D} \exp\left(-\frac{(I - I_0)^2}{2\sigma_1^2}\right) dI = \frac{1}{2}\,\mathrm{erfc}\left(\frac{I_1 - I_D}{\sigma_1\sqrt{2}}\right)$$

$$P(1/0) = \frac{1}{\sigma_0\sqrt{2\pi}} \int_{I_D}^{\infty} \exp\left(-\frac{(I - I_0)^2}{2\sigma_0^2}\right) dI = \frac{1}{2}\,\mathrm{erfc}\left(\frac{I_D - I_0}{\sigma_0\sqrt{2}}\right)$$

where $\mathrm{erfc} = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-y^2}\, dy$

The BER becomes

$$BER = \frac{1}{4}\left[\mathrm{erfc}\left(\frac{I_1 - I_D}{\sigma_1\sqrt{2}}\right) + \mathrm{erfc}\left(\frac{I_D - I_0}{\sigma_0\sqrt{2}}\right)\right]$$

Bit Error Rate

The decision threshold I_D is optimized to minimize BER
I_D is chosen such that

$$\frac{(I_D - I_0)^2}{2\sigma_0^2} = \frac{(I_1 - I_D)^2}{2\sigma_1^2} + \ln\left(\frac{\sigma_1}{\sigma_0}\right)$$

the ln() term is usually negligible so

$$I_D = \frac{\sigma_0 I_1 + \sigma_1 I_0}{\sigma_0 + \sigma_1}$$

If thermal noise dominates $\sigma_0 = \sigma_1$ and $I_D = \frac{I_0 + I_1}{2}$

The BER is $BER = \frac{1}{2}\,\mathrm{erfc}\left(\frac{Q}{\sqrt{2}}\right)$ where $Q \equiv \frac{I_1 - I_0}{\sigma_1 + \sigma_0}$

for large Q ($Q > 3$)

$$BER = \frac{\exp(-Q^2/2)}{Q\sqrt{2\pi}}$$

for $BER = 10^{-9}$ $Q = 6$

Use the $Q = 6$ to calculate the minimum detectable power

$I_1 = MRP_1 = 2MRP_{rec}$

P_{rec} is the average power $P_{rec} = \frac{P_1 + P_0}{2}$

$I_0 = 0$

$\sigma_0 = \sigma_{th}$ $\sigma_1 = \sqrt{\sigma_{sh}^2 + \sigma_{th}^2}$

$$P_{rec} = \frac{Q}{R}\left(q F_A Q \Delta f + \frac{\sigma_T}{M}\right)$$

For a thermal noise limited receiver

$$P_{rec} = \frac{Q}{R}\frac{\sigma_T}{M}$$

There are other noise contributions such as:

- Extinction ratio
- Relative intensity noise
- timing Jitter
- etc.