In some materials the crystal symmetry causes

$$r = 0$$

$$n(\vec{E}) = n_0 + [S]\vec{E}_1 \vec{E}_2$$

This is the quadratic electro-optic effect and is usually substantially weaker than the linear EO effect

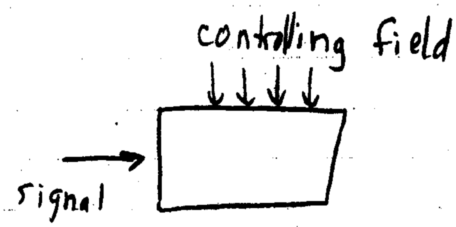
We will concentrate on the linear EO effect

$$\Delta n = n(\vec{E}) - n_0 = \Delta\left(\frac{1}{n^2}\right) = [r]\,\vec{E}$$

There is a difference between the applied field and the signal field

$$\Delta n = \bar{\bar{r}}\,\underbrace{\bar{E}}_{\text{applied field}}$$

$$\vec{D} = [\epsilon]\,\vec{E}$$



The nonlinear effect relates the index of refraction of a material to the intensity of applied field

All materials are nonlinear
Some materials have substantially higher non-linear coefficients

$$n(E) \approx n + a_1 E + a_2 E^2 + \cdots$$

It is more common to use the impermeability

$$\eta = \frac{\epsilon_0}{\epsilon} = \frac{1}{n^2}$$

$$\eta(E) = \eta_0 + rE + SE^2 + \cdots$$

In many practical cases

$$rE \gg SE^2$$

$$\eta(\vec{E}) = \eta_0 + [r]\vec{E}$$

This is called the linear electro-optic effect

r is called the Pockels coefficient

A uniaxial birefringent crystal has an index ellipsoid represented by

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_E^2} = 1$$

An applied electric field changes the dimensions and orientation of the index ellipsoid. The ellipsoid becomes

$$A x^2 + B y^2 + C z^2 + D\, yz + E\, xz + F\, xy = 1$$

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + 2\left(\frac{1}{n^2}\right)_4 yz + 2\left(\frac{1}{n^2}\right)_5 xz + 2\left(\frac{1}{n^2}\right)_6 xy = 1$$

$$\Delta\left(\frac{1}{n^2}\right)_i = \sum_j r_{ij} E_j \qquad \begin{array}{l} j = x, y, z \\ i = 1, \dots 6 \end{array}$$

$$\Delta\left(\frac{1}{n^2}\right)_{ij} = \sum_K r_{ijK} E_K$$

By tensor symmetry

$$r_{ijK} = r_{jiK}$$

So
$$\begin{array}{lll}
ij = 1\ 1 & \Rightarrow & i = 1 \\
ij = 2\ 2 & \Rightarrow & i = 2 \\
ij = 3\ 3 & \Rightarrow & i = 3 \\
ij = 23,\ 32 & \Rightarrow & i = 4 \\
ij = 13,\ 31 & \Rightarrow & i = 5 \\
ij = 12,\ 21 & \Rightarrow & i = 6
\end{array}$$

$$r_{ijk} \Rightarrow r_{ik}$$

$$\begin{bmatrix} \Delta(1/n^2)_1 \\ \Delta(1/n^2)_2 \\ \Delta(1/n^2)_3 \\ \Delta(1/n^2)_4 \\ \Delta(1/n^2)_5 \\ \Delta(1/n^2)_6 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

The electro-optic tensor $[r]$ is represented by a 6×3 matrix. The crystal symmetry determines which coefficients are zero but not the magnitude.

FORM OF THE LINEAR ELECTROOPTIC TENSOR

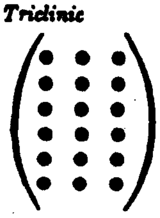
Symbols:

- zero element
- nonzero element
- equal nonzero elements
- equal nonzero elements, but opposite in sign

The symbol at the upper left corner of each tensor is the conventional symmetry group designation.

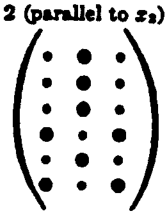
Centrosymmetric—All elements zero

Triclinic

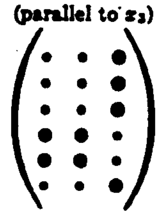


Monoclinic

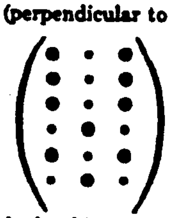
2 (parallel to x_2)



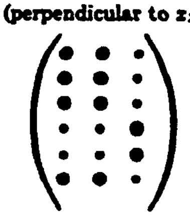
(parallel to x_3)



m (perpendicular to x_2)

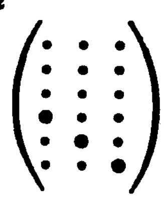


(perpendicular to x_3)

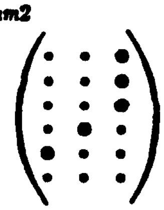


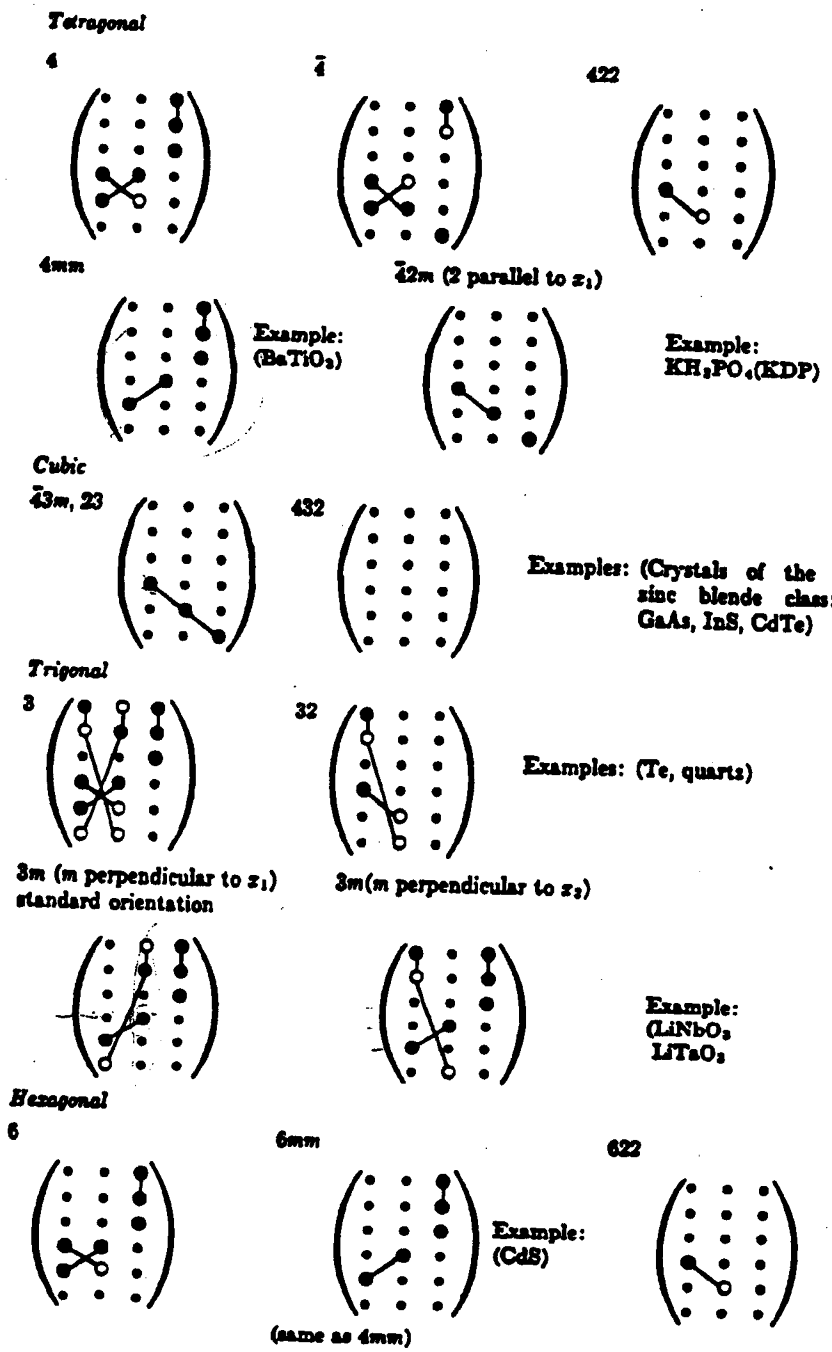
Orthorhombic

222



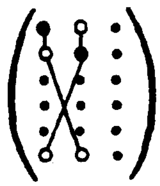
mm2



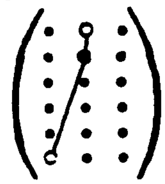

Tetragonal
4
$\bar{4}$
422
4mm
Example: (BaTiO$_3$)
$\bar{4}2m$ (2 parallel to x_1)
Example: KH$_2$PO$_4$(KDP)
Cubic
$\bar{4}3m$, 23
432
Examples: (Crystals of the zinc blende class: GaAs, InS, CdTe)
Trigonal
3
32
Examples: (Te, quartz)
3m (m perpendicular to x_1) standard orientation
3m(m perpendicular to x_2)
Example: (LiNbO$_3$ LiTaO$_3$
Hexagonal
6
6mm
Example: (CdS)
622
(same as 4mm)

24

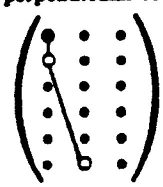
$\bar{6}$



$\bar{6}m2$ (*m* perpendicular to x_1 standard orientation)



(*m* perpendicular to x_2)



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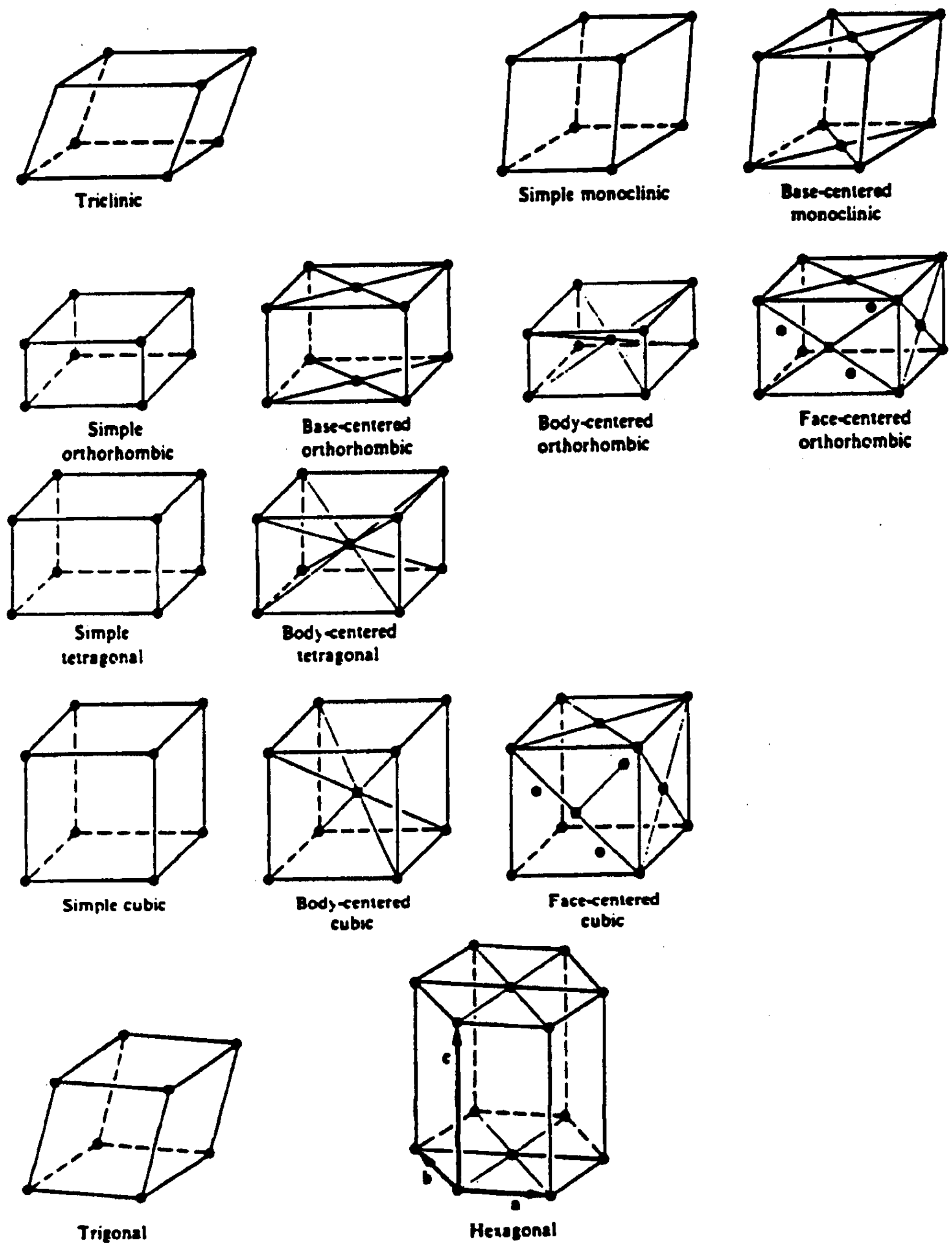
SEVEN CRYSTAL SYSTEMS
(Pictorial Representation)



metry class]. The electro-optic tensor has the following form (contracted notation):

$$\begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{13} & 0 \\ r_{13} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

V. Waveguide Properties of Nonlinear Optical Polymers

If a nonlinear optical polymer is to be used for an integrated optic modulator or switch, it also needs to satisfy a number of basic properties for waveguiding; these are appropriate confinement in transversal and lateral direction as well as low absorption. In terms of these properties, nonlinear optical polymer waveguides are rather similar to passive polymer waveguides. The first reports on passive waveguides in organic thin films, and comparative results of several materials and techniques, date from the early 1970's [37], [38], [49]–[52]. Losses well below 1 dB/cm (best value: 0.04 dB/cm) have been measured for the fundamental modes in slab waveguides. For nonlinear optical polymer waveguides, similar results have been reported recently, although absorption levels are generally higher than for passive polymers. This is caused by the highly nonlinear organic compounds which are strongly colored and absorb at shorter wavelengths [19].

The transversal confinement in polymeric waveguides is generally obtained by using a stacked multilayer of different polymers or materials such as SiO_2, Si_3N_4, or SiO_xN_y [6] with different refractive index, or by using modified versions of the same polymer. A technological problem can be that subsequent polymeric layers dissolve each other or cause cracks.

To obtain lateral confinement for waveguides in thin films, different techniques exist. The differences are mostly imposed by specific material properties. Standard techniques like dry etching [2], [33] can be used to fabricate standard configurations like rib waveguides, strip waveguides, or inverted rib waveguides. Losses around 6 dB/cm have been reported for inverted rib polymeric waveguides (monomodal operation at 1300 nm) fabricated on top of a GaAs or Si substrate covered by a PECVD SiO_2-layer [6], [53] or below 3 dB/cm using a UV curable epoxy as cladding layers [54]. Other more complex schemes use local ion exchange in low index glass to achieve an inverted strip loaded guide in the polymer [55] and wet processing of polymer gelatin [56], [57], or solvent-assisted indiffusion of nonlinear molecules to obtain graded index guides, with mode profiles compatible with graded index fibers [58], [59]. Losses of a few dB/cm are reported. In all these experiments, it is not always clear to what extent the losses are due to either absorption or scattering.

The most promising and flexible technique for obtaining lateral confinement, however, makes use of chemical transformations induced by UV irradiation. This exposure can increase

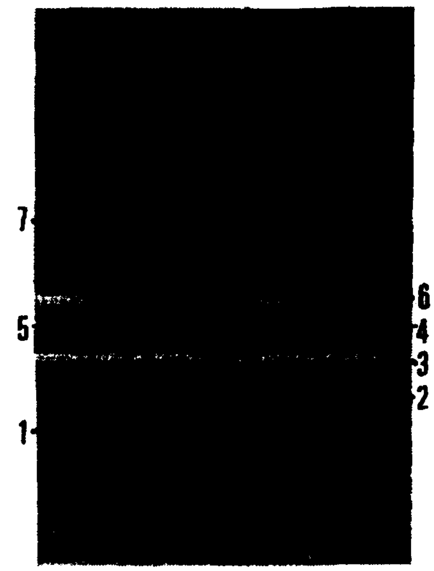


Fig. 2. Microscope picture (with interference contrast) of endface of a polymeric waveguide layer structure. The scale glass substrate, (2) the uniform underelectrode, (3) the underc (4) the bleached core layer, (5) the waveguide (unbleached), cladding and the Au stripes used for bleaching, and (7) UV

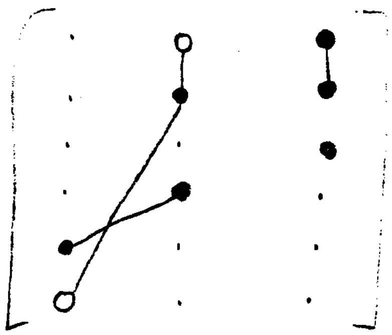
nique suitable for mass production were already de [65]. Other techniques include direct laser writin tooxidation of polyalkylsilines (decreasing the refra [67], "photolocking" (locking of dopants in a pol under influence of UV) [49], [68], and diffusion of ular weight constituents into local polymerizing re [70].

In the framework of the European RACE 1((Polymeric Optical Switches), a sidechain polymer f AKZO is used at our laboratory [71]. The structure chains, highly polarizable DANS molecules, can be UV-exposure allowing a decrease in refractive inde Waveguides are now formed by exposing the poly through a mask. The result is a completely planar s extremely smooth waveguides and very low scatte This process also allows us to tune devices, fabrica parent substrates, after complete fabrication. Usi bleaching, waveguides have been fabricated with l than 1 dB/cm at 1.3 μm [71]. A similar technique in [74], with relatively high waveguide losses, hov

Although, in comparison to III–V materials and l technology for fabricating waveguides in polymers these results are already quite reasonable. For InP–l waveguides, typical losses are below 1 dB/cm at λ but values below 0.2 dB/cm have already been re [77]. For Ti-diffused waveguides in $LiNbO_3$, losse lower: typical results are around 0.3 dB/cm at λ [78], with best results down to 0.1 dB/cm [79]. fractive index of polymers (1.6–1.7) does, in prin tate coupling of light in or out of a waveguide. however, some extra problems have to be solved si along crystallographic planes (as in III–V) is not p tially, prism coupling was used to excite the diff modes in the film [37], [38], [51], but the risk of d

Example $LiNbO_3$

Trigonal $3m$

 $= \begin{bmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{bmatrix}$

see Table 14.4 in book page 701

$r_{13} = 9.6$
$r_{22} = 6.8$
$r_{33} = 30.9$
$r_{51} = 32.6$
$\times 10^{-12}$ m/V

$$\Delta(1/n^2)_1 = -r_{22} E_y + r_{13} E_z$$
$$\Delta(1/n^2)_2 = r_{22} E_y + r_{13} E_z$$
$$\Delta(1/n^2)_3 = r_{33} E_z$$
$$\Delta(1/n^2)_4 = r_{51} E_y$$
$$\Delta(1/n^2)_5 = r_{51} E_x$$
$$\Delta(1/n^2)_6 = -r_{22} E_x$$

The index ellipsoid becomes

$$\left(\frac{1}{n_o^2} - r_{22} E_y + r_{13} E_z\right)x^2 + \left(\frac{1}{n_o^2} + r_{22} E_y + r_{13} E_z\right) y^2$$

$$+\left(\frac{1}{n_E^2} + r_{33} E_z\right) z^2 + 2 r_{51} E_y yz + 2 r_{51} E_x xz$$

$$-2 r_{22} E_x xy = 1$$

With no applied field this reduces to

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_E^2} = 1$$

A rotation of the axes are require to transform the equation into the form

$$\frac{(x')^2}{(n_{x'})^2} + \frac{(y')^2}{(n_{y'})^2} + \frac{(z')^2}{(n_{z'})^2} = 1$$

Lithium Niobate with $E = E_z$

$$\left(\frac{1}{n_o^2} + r_{13} E_z\right)x^2 + \left(\frac{1}{n_o^2} + r_{13} E_z\right)y^2 + \left(\frac{1}{n_E^2} + r_{33} E_z\right)z^2 = 1$$

No rotation of the ellipsoid
Still uniaxial

$$\frac{1}{(n_x')^2} = \frac{1}{(n_y')^2} = \frac{1}{n_o^2} + r_{13} E_z$$

$$= \frac{1}{n_o^2}\left(1 + n_o^2 r_{13} E_z\right)$$

$$n_x' = n_y' = n_o\left(1 + n_o^2 r_{13} E_z\right)^{-1/2} \qquad n_o^2 r_{13} E_z \ll 1$$

$$n_x' = n_y' = n_o\left(1 - \tfrac{1}{2} n_o^2 r_{13} E_z\right)$$

$$= n_o - \frac{n_o^3}{2} r_{13} E_z$$

$$n_{x'} = n_o - \frac{n_o^3}{2} r_{13} E_z$$

$$n_{y'} = n_o - \frac{n_o^3}{2} r_{13} E_z$$

$$n_z' = n_E - \frac{n_E^3}{2} r_{33} E_z$$

Propagation in x-direction

polarization in y-direction : $n = n_o - \frac{n_o^3}{2} r_{13} E_z$

polarization in z-direction $n = n_E - \frac{n_E^3}{2} r_{33} E_z$

$$\Delta n = n_o - n_E - \left(\frac{n_o^3}{2} r_{13} - \frac{n_E^3}{2} r_{33} \right) E_z$$

with $E = E_y$

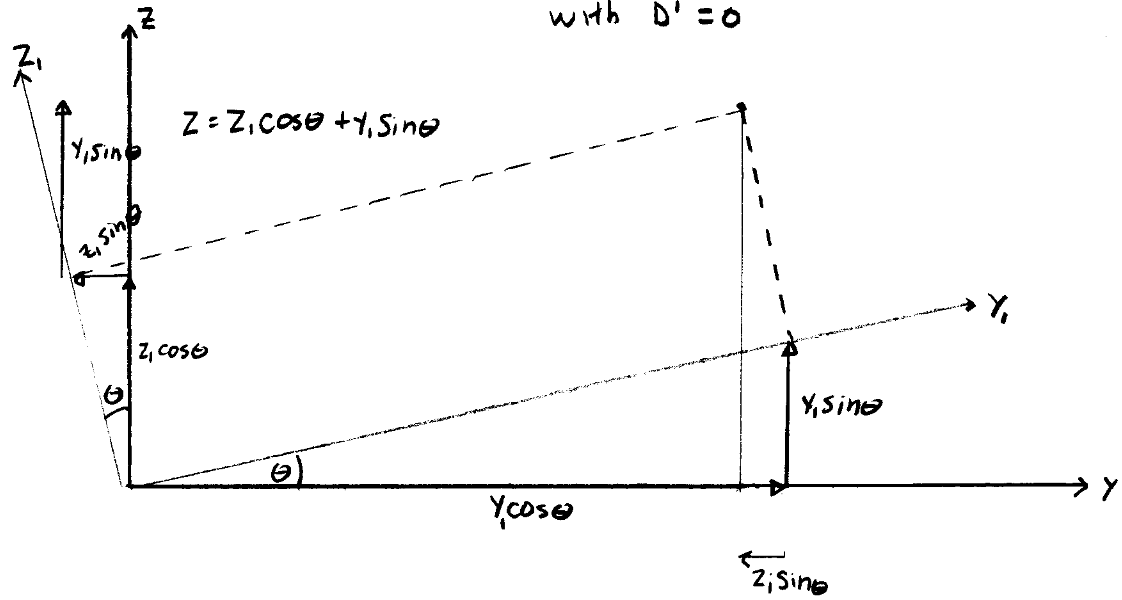
$$\left(\frac{1}{n_o^2} - r_{22} E_y\right) x^2 + \left(\frac{1}{n_o^2} + r_{22} E_y\right) y^2 + \frac{1}{n_E^2} E_z + 2\, r_{42} E_y\, yz = 1$$

the yz term indicates a rotation about the x-axis

The present equation $Ax^2 + By^2 + Cz^2 + Dyz = 1$ need to become

$$A'x_1^2 + B'y_1^2 + C'z_1^2 + D'y_1 z_1 = 1$$

with $D' = 0$



$Y = Y_1 \cos\theta - Z_1 \sin\theta$

$X = x_1$
$Y = Y_1 \cos\theta - Z_1 \sin\theta$
$Z = Y_1 \sin\theta + Z_1 \cos\theta$

$x^2 = x_1^2$
$y^2 = y_1^2 \cos^2\theta - 2 y_1 z_1 \sin\theta\cos\theta + z_1^2 \sin^2\theta$
$z^2 = y_1^2 \sin^2\theta + 2 y_1 z_1 \sin\theta\cos\theta + z_1^2 \cos^2\theta$
$yz = y_1^2 \sin\theta\cos\theta + y_1 z_1 (\cos^2\theta - \sin^2\theta) - z_1^2 \sin\theta\cos\theta$

$$\begin{aligned} A(x_1^2) &+ B\left(y_1^2 \cos^2\theta - 2 y_1 z_1 \sin\theta\cos\theta + z_1^2 \sin^2\theta\right) \\ &+ C\left(y_1^2 \sin^2\theta + 2 y_1 z_1 \sin\theta\cos\theta + z_1^2 \cos^2\theta\right) \\ &+ D\left(y_1^2 \sin\theta\cos\theta + y_1 z_1 (\cos^2\theta - \sin^2\theta) - z_1^2 \sin\theta\cos\theta\right) = 1 \end{aligned}$$

$$\begin{aligned} x_1^2(A) &+ y_1^2\left(B\cos^2\theta + C\sin^2\theta + D\sin\theta\cos\theta\right) \\ &+ z_1^2\left(B\sin^2\theta + C\cos^2\theta - D\sin\theta\cos\theta\right) \\ &+ y_1 z_1\left(-2B\sin\theta\cos\theta + 2C\sin\theta\cos\theta + D(\cos^2\theta - \sin^2\theta)\right) = 1 \end{aligned}$$

$A' = A$

$B' = B\cos^2\theta + C\sin^2\theta + D\sin\theta\cos\theta$

$C' = B\sin^2\theta + C\cos^2\theta - D\sin\theta\cos\theta$

$D' = -2B\sin\theta\cos\theta + 2C\sin\theta\cos\theta + D(\cos^2\theta - \sin^2\theta)$

Using the identities

$$2\sin\theta\cos\theta = \sin 2\theta$$
$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$D' = -B\sin 2\theta + C\sin 2\theta + D\cos 2\theta$$

set $D' = 0$

$$0 = (B-C)\sin 2\theta - D\cos 2\theta$$

$$\tan 2\theta = \frac{D}{B-C}$$

$$\tan 2\theta = \frac{2r_{51}E_y}{\frac{1}{n_o^2} - \frac{1}{n_E^2} + r_{22}E_y}$$

from Table 14.4

$$\frac{1}{n_o^2} - \frac{1}{n_E^2} = -.015$$
$$r_{22}E_y = 6.8\times10^{-12}\ E_y$$
$$2r_{51}E_y = 65.2\times10^{-12}\ E_y$$

So $|1/n_o^2 - 1/n_E^2| \gg r_{22}E_y,\ 2r_{51}E_y$ for $E_y < 10^4$ V/cm

So θ is very small

$$A' \approx A$$
$$B' \approx B$$
$$C' \approx C$$

The ellipsoid is

$$\left(\frac{1}{n_o^2} - r_{22} E_y\right) x_1^2 + \left(\frac{1}{n_o^2} + r_{22} E_y\right) y_1^2 + \left(\frac{1}{n_E^2}\right) z_1^2 = 1$$

$$n_{x_1} \simeq n_o + \frac{n_o^3}{2} r_{22} E_y$$

BIAXIAL CRYSTAL

$$n_{y_1} \simeq n_o - \frac{n_o^3}{2} r_{22} E_y$$

$$n_{z_1} \simeq n_E$$

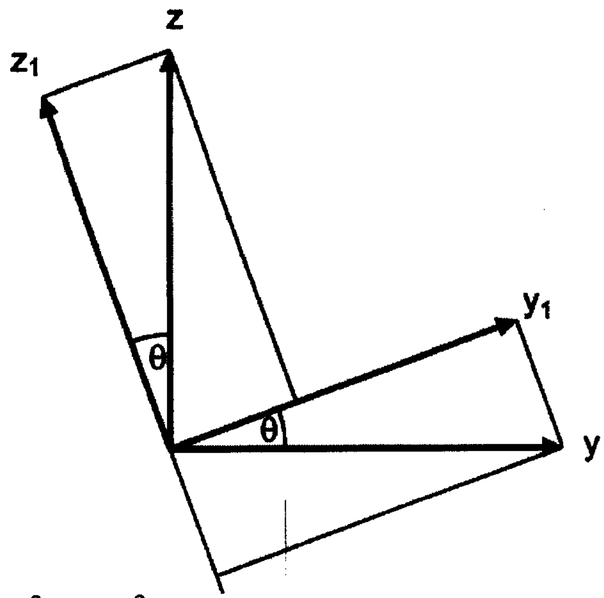
Now for $E = E_x$

$$\frac{1}{n_o^2} x^2 + \frac{1}{n_o^2} y^2 + \frac{1}{n_E^2} z^2 + 2 r_{51} E_x xz - 2 r_{22} E_x xy = 1$$

This requires a rotation about y and a rotation about z

We will not go through this. But, the rotation about the y-axis is small because of the large natural birefringence. The rotation about z is not small since there is no natural birefringence.

Axis Rotation



$x = x_1$
$y = y_1 \cos\theta - z_1 \sin\theta$
$z = y_1 \sin\theta + z_1 \cos\theta$

$x^2 = (x_1)^2$
$y^2 = (y_1)^2 \cos^2\theta - 2\, y_1\, z_1 \sin\theta \cos\theta + (z_1)^2 \sin^2\theta$
$z^2 = (y_1)^2 \sin^2\theta - 2\, y_1\, z_1 \sin\theta \cos\theta + (z_1)^2 \cos^2\theta$
$yz = (y_1)^2 \sin\theta \cos\theta - 2\, y_1\, z_1 (\cos^2\theta - \sin^2\theta) - (z_1)^2 \sin\theta \cos\theta$

$A\,(x_1^2) + B\,(y_1^2 \cos^2\theta - 2\, y_1\, z_1 \sin\theta \cos\theta + z_1^2 \sin^2\theta)$
$+ C\,(y_1^2 \sin^2\theta - 2\, y_1\, z_1 \sin\theta \cos\theta + z_1^2 \cos^2\theta)$
$+ D\,(y_1^2 \sin\theta \cos\theta - 2\, y_1\, z_1 (\cos^2\theta - \sin^2\theta) - z_1^2 \sin\theta \cos\theta) = 1$

$A_1 = A$
$B_1 = B \cos^2\theta + C \sin^2\theta + D \sin\theta \cos\theta$
$C_1 = B \sin^2\theta + C \cos^2\theta - D \sin\theta \cos\theta$
$D_1 = -2B \sin\theta \cos\theta + 2C \sin\theta \cos\theta + D (\cos^2\theta - \sin^2\theta)$

If the axis rotation is xy replace z with x and C with A
$C_1 = C$
$B_1 = B \cos^2\theta + A \sin^2\theta + D \sin\theta \cos\theta$
$A_1 = B \sin^2\theta + A \cos^2\theta - D \sin\theta \cos\theta$
$D_1 = -2B \sin\theta \cos\theta + 2A \sin\theta \cos\theta + D (\cos^2\theta - \sin^2\theta)$

If the axis rotation is xz replace y with x and B with A
$B_1 = B$
$A_1 = A \cos^2\theta + C \sin^2\theta + D \sin\theta \cos\theta$
$C_1 = A \sin^2\theta + C \cos^2\theta - D \sin\theta \cos\theta$
$D_1 = -2A \sin\theta \cos\theta + 2C \sin\theta \cos\theta + D (\cos^2\theta - \sin^2\theta)$

Example Done in Class
GaAs Modulator

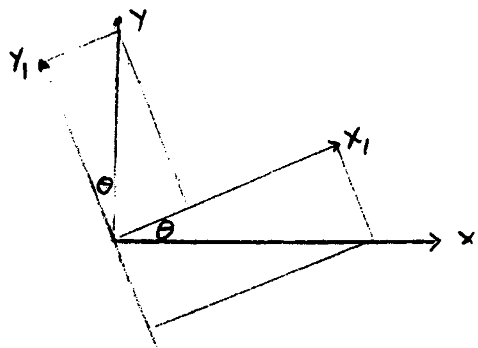
cubic $\bar{4}3m$
isotropic $n = 3.6$
$r_{41} = 1.1$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{41} \end{bmatrix}$$

$E = E_z$

$$\left(\frac{1}{n^2}\right)x^2 + \left(\frac{1}{n^2}\right)y^2 + \left(\frac{1}{n^2}\right)z^2 + 2 r_{41} E_z \, x y = 1$$

rotate about the z-axis



$$x = x_2 \cos\theta - y_2 \sin\theta$$
$$y = x_2 \sin\theta + y_2 \cos\theta$$
$$z = z_2$$

$$x^2 = x_2^2 \cos^2\theta - 2 x_2 y_2 \sin\theta \cos\theta + y_2^2 \sin^2\theta$$
$$y^2 = x_2^2 \sin^2\theta + 2 x_2 y_2 \sin\theta \cos\theta + y_2^2 \cos^2\theta$$
$$z^2 = z_2^2$$
$$xy = x_2^2 \sin\theta\cos\theta + x_2 y_2 (\cos^2\theta - \sin^2\theta) - y_2^2 \sin\theta\cos\theta$$

$$2\sin\theta\cos\theta = \sin 2\theta$$
$$\cos^2\theta - \sin^2\theta = \cos 2\theta$$

$$A' = A\cos^2\theta + B\sin^2\theta + D\sin\theta\cos\theta$$
$$B' = A\sin^2\theta + B\cos^2\theta - D\sin\theta\cos\theta$$
$$C' = C$$
$$D' = -2A\sin\theta\cos\theta + 2B\sin\theta\cos\theta + D(\cos^2\theta - \sin^2\theta)$$

set $D' = 0$

$$D' = -A \sin 2\theta + B \sin 2\theta + D \cos 2\theta = 0$$

$$\tan 2\theta = \frac{D}{A-B}$$

$$\tan 2\theta = \frac{2 r_{41} E_z}{\frac{1}{n^2} - \frac{1}{n^2}} = \infty$$

$$\theta = 45^\circ \qquad \cos^2\theta = \sin^2\theta = \sin\theta\cos\theta = \tfrac{1}{2}$$

$$A' = \tfrac{1}{2} A + \tfrac{1}{2} B + \tfrac{1}{2} D$$
$$= \tfrac{1}{2}\left(\frac{1}{n^2} + \frac{1}{n^2} + 2 r_{41} E_z\right)$$
$$= \frac{1}{n^2} + r_{41} E_z$$

$$B' = \tfrac{1}{2}\left(\frac{1}{n^2} + 1/n^2 - 2 r_{41} E_z\right)$$
$$= 1/n^2 - r_{41} E_z$$

$$C' = 1/n^2$$

$$\frac{1}{(n_x')^2} = \frac{1}{n^2} + r_{41} E_z$$

$$n_x' = n + \frac{n^3}{2} r_{41} E_z$$

$$n_y' = n - \frac{n^3}{2} r_{41} E_z$$

$$n_z = n$$

2 Forms of phase delay

(1) Absolute phase delay
(2) Relative phase delay between polarizations

Let's look at the relative phase delay

For a GaAs modulator with $E = E_z$

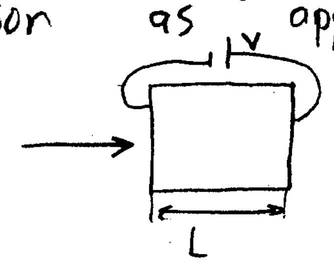
$$n_x' = n + \tfrac{1}{2} n^3 r_{41} E_z$$
$$n_y' = n - \tfrac{1}{2} n^3 r_{41} E_z$$
$$n_z' = n$$

What is the relative phase delay for different propagation directions

Maximum Δn is in the z'-direction

$$\Delta n = n^3 r_{41} E_z$$

This is propagation in the same direction as applied field



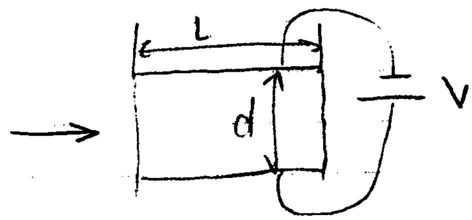
$$\Gamma = \frac{2\pi}{\lambda} \Delta n L = \frac{2\pi}{\lambda} L \, n^3 r_{41} E_z$$
$$= \frac{2\pi}{\lambda} n^3 r_{41} L \frac{V}{L} = \frac{2\pi}{\lambda} n^3 r_{41} V$$

This is called a longitudinal modulator

for $\Gamma = \pi$

$V = \frac{\lambda}{2 n^3 r_{41}}$ $\quad V = 15.9$ KV

Now lets have propagation in th y' direction



$\Delta n = \frac{1}{2} n^3 r_{41} E_z$

$\Gamma = \frac{2\pi}{\lambda} L \; \frac{1}{2} n^3 r_{41} \; E_z$

$= \frac{\pi}{\lambda} n^3 \; r_{41} \; L \frac{V}{d}$

$\Gamma = \frac{\pi}{\lambda} n^3 \; r_{41} \; \frac{L}{d} V$

This is called a transverse modulator

for $\Gamma = \pi$

longitudinal $\quad V = \frac{\lambda}{2 n^3 r_{41}}$

transverse $\quad V = (\lambda) \left(\frac{d}{L}\right) \frac{1}{n^3 r_{41}}$

for $r_{41} = 1.1 \times 10^{-17}$ m/V
$\lambda = 0.9$ um

longitudinal $\quad V = 8.8$ KV

trasverse $\quad V = 17.6 \times 10^3 \left(\frac{d}{L}\right) V \quad$ with $d = .5$um $\quad L = 1000$um
$V = 8.8$ V

The ~~[illegible]~~ most common external modulator is $LiNbO_3$. But GaAs is easier to fabricate. Lets look at the modulation

For a longitudinal modulator

GaAs : $\vec{E} = E_z$ $\quad \Delta n = n^3 r_{41} E_z$
$\vec{\beta} = \beta_y$

for $LiNbO_3$: $\vec{E} = E_z$ $\quad \Delta n = \underbrace{n_O - n_E}_{\text{ignore fixed } \beta} - \left(\frac{n_O^3}{2} r_{13} - \frac{n_E^3}{2} r_{33} \right) E_z$
$\vec{\beta} = \beta_y$

$\vec{E} = E_y$
$\vec{\beta} = \beta_z$ $\quad \Delta n = n_O^3 \, r_{22} \, E_y$

GaAs : $r_{41} = 1.4$

$LiNbO_3$: $r_{13} = 8.6$ $\quad r_{33} = 30.8$ $\quad r_{22} = 3.4$

Material	$\Delta n / E \times 10^{-12}$
GaAs	51.3
$LiNbO_3$ (E_z)	112.6
$LiNbO_3$ (E_y)	40.6

So best to use Z-cut $LiNbO_3$

We will call $\left(r_{33} - r_{13} \frac{n_O^3}{n_E^3} \right) = r_c$

Let's look at GaAs

This is cubic $\bar{4}3m$

$$r = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{41} \end{bmatrix}$$

There are only cross terms

GaAs is initially isotropic

There is no difference in direction
If we use E_z

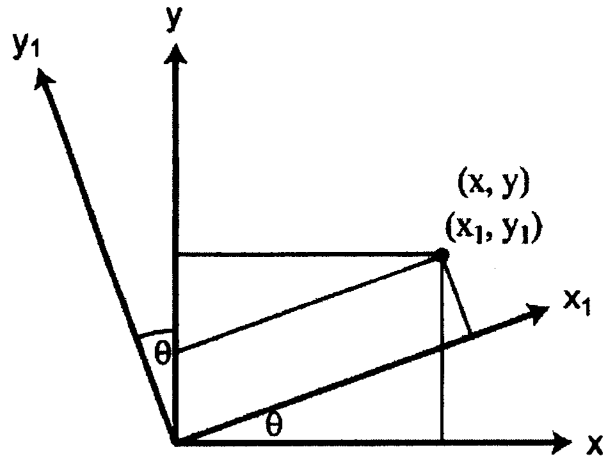
$\Delta \eta_1 \rightarrow \Delta \eta_5 = 0$
$\Delta \eta_6 = 2 r_{41} E_z$

The index ellipsoid becomes

$$\frac{1}{n^2} x^2 + \frac{1}{n^2} y^2 + \frac{1}{n^2} z^2 + 2 r_{41} E_z \, xy = 1$$

We need to rotate the axis to elliminate the xy term

Axis Rotation



After rotating the coordinate axis the point (x_1, y_1) becomes (x,y)

$$x = x_1 \cos\theta - y_1 \sin\theta$$
$$y = x_1 \sin\theta + y_1 \cos\theta$$
$$z = z_1$$

$$x^2 = (x_1)^2 \cos^2\theta - 2\, x_1\, y_1 \sin\theta \cos\theta + (y_1)^2 \sin^2\theta$$
$$y^2 = (x_1)^2 \sin^2\theta + 2\, x_1\, y_1 \sin\theta \cos\theta + (y_1)^2 \cos^2\theta$$
$$z^2 = (z_1)^2$$
$$xy = (x_1)^2 \sin\theta \cos\theta + 2\, x_1\, y_1 (\cos^2\theta - \sin^2\theta) - (y_1)^2 \sin\theta \cos\theta$$

$$Ax^2+By^2+Cy^2+Fxy=1$$

Replace x, y, and z to determine the new index ellipsoid

$$A\,(x_1^2 \cos^2\theta - 2\, x_1\, y_1 \sin\theta \cos\theta + y_1^2 \sin^2\theta)$$
$$+ B\,(x_1^2 \sin^2\theta + 2\, x_1\, y_1 \sin\theta \cos\theta + y_1^2 \cos^2\theta)$$
$$+ C\, z_1^2$$
$$+ F\,(x_1^2 \sin\theta \cos\theta + 2\, x_1\, y_1 (\cos^2\theta - \sin^2\theta) - y_1^2 \sin\theta \cos\theta) = 1$$

Combine the x_1, y_1, and z_1 terms

$$x_1^2\,(A \cos^2\theta + B \sin^2\theta + F \sin\theta \cos\theta)$$
$$+ y_1^2\,(A \sin^2\theta + B \cos^2\theta - F \sin\theta \cos\theta)$$
$$+ z_1^2\,(F)$$
$$+ x_1\, y_1\,(-2A \sin\theta \cos\theta + 2B \sin\theta \cos\theta + 2F (\cos^2\theta - \sin^2\theta)) = 1$$

$$A_1 = A \cos^2\theta + B \sin^2\theta + F \sin\theta \cos\theta$$
$$B_1 = A \sin^2\theta + B \cos^2\theta - F \sin\theta \cos\theta$$
$$C_1 = C$$
$$F_1 = -2 \sin\theta\cos\theta\, (A - B) + F\,(\cos^2\theta - \sin^2\theta)$$

set $F_1 = 0$ to elliminate the cross-term

$$-2 \sin\theta \cos\theta (A-B) + F(\cos^2\theta - \sin^2\theta) = 0$$

using trig identities

$$-(A-B)\sin 2\theta + F(1 - 2\sin^2\theta) = 0$$

$$-(A-B)\sin 2\theta + F(\cos 2\theta) = 0$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{A-B}{F}$$

Now go back to the original index ellipsoid equation

$$\frac{1}{n^2}x^2 + \frac{1}{n^2}y^2 + \frac{1}{n^2}z^2 + 2r_{41}E_z xy = 1$$

$A = B = C = \frac{1}{n^2}$

$F = 2r_{41}E_z$

$$2\theta = \tan^{-1}\left(\frac{0}{2r_{41}E_z}\right) = 90°$$

$\theta = 45°$

$A' = \frac{1}{2}A + \frac{1}{2}B + \frac{1}{2}F$

$B' = \frac{1}{2}A + \frac{1}{2}B - \frac{1}{2}F$

$C = C$

$A' = \frac{1}{n^2} + r_{41}E_z$

$B' = \frac{1}{n^2} - r_{41}E_z$

$C' = \frac{1}{n^2}$

$$n_x = n - \frac{1}{2} n^3 r_{41} E_z$$
$$n_y = n + \frac{1}{2} n^3 r_{41} E_z$$
$$n_z = n$$

best propagation direction is $\hat{z}$

Birefringence: $B = n^3 r_{41} E_z$

$$\Gamma = n^3 r_{41} E_z \frac{2\pi}{\lambda} L$$