Optical amplifiers add noise called amplified spontaneous emission (ASE) noise

The power spectral density is:

$$S_{ASE}(f) = (G-1)\, n_{sp}\, \chi\, hf$$

$$n_{sp} = \frac{N_2}{N_2 - N_1}$$

$\chi > 1$ accounts for non-uniform carrier density

For large gain, the dominant ASE noise term is

$$\sigma^2_{sig-ASE} = 4(R G P_{in})(R S_{ASE} B)$$

$$= 4(R G P_{in})(R [G-1] n_{sp} \chi hf) B$$

for $G \gg 1$

$$\sigma^2_{ASE} = 4(R G P_{in})(R G\, n_{sp} \chi hf) B$$

$$SNR = \frac{[R P_{in} G]^2}{[(R\,NEP)^2 + 2q R P_{in} + 4 R^2 G^2 P_{in} n_{sp} \chi hf] B}$$

Let's look at the limits:

If no EDFA is used ($G = 1$)

$$SNR = \frac{(R P_{in})^2}{[(R NEP)^2 + 2q R P_{in}] B}$$

Same as before

If large G: $4R^2 G^2 P_{in} n_{sp} \chi hf \gg (R NEP)^2$
$\gg 2q R P_{in}$

$$SNR = \frac{(R P_{in} G)^2}{(4 R^2 G^2 P_{in} n_{sp} \chi hf) B}$$

$$SNR = \frac{P_{in}}{4 n_{sp} \chi hf B}$$

SNR decreases with received power

$$SNR = \frac{N_b}{4 n_{sp} \chi}$$

$$N_b = \frac{P_{in}}{hf B}$$

Number of photons per bit

Example : $R = 0.8\ A/W$
$P_{tx} = 10\ mW$
$B = 1\ GHz$
$SNR \geq 10$
$NEP = 10^{-13}\ W/Hz^{1/2}$
$\alpha_{fiber} = 0.2\ dB/km$

(a) What is the maximum transmission distance?
(Ignore all losses except fiber attenuation)
Find P_{min}
Assume that $R\,NEP \gg 2qRP_{in}$

$$10 = \frac{(RP_{in})^2}{(R\,NEP)^2 B} = \frac{P_{in}^2}{NEP^2 B}$$

$$P_{in} = \sqrt{(10)(10^{-13})^2\, 10^9} = 10\ nW$$

$P_{tx} = 10\ mW = 10\ dBm$
$P_{min} = 10\ nW = -50\ dBm$

$PB = 60\ dB = 0.2\ L$

$L = 300\ km$

Now use an EDFA with
$G = 30\,dB$ $\quad n_{sp}\chi = 10$
$P_{sat} = 10\,mW$

Where is the EDFA placed?

If it is placed at the transmitter it will not do anything

Place it at the receiver

$$SNR = \frac{(R P_{in} G)^2}{[(R\,NEP)^2 + 4 R^2 G^2 P_{in} n_{sp}\chi h f] B} = 1.89$$

drop in SNR

$$4 R^2 G^2 P_{in} n_{sp} \chi h f \gg (R\,NEP)^2$$

$$SNR = \frac{(R P_{in} G)^2}{4 R^2 G^2 P_{in} n_{sp}\chi h f B} = \frac{P_{in}}{4 n_{sp}\chi h f B}$$

Place the EDFA in the middle of the link

$P_{mid} = 10\,dBm - (150)(0.2) = -20dB = 10\mu W$

$\sigma^2_{ASE} = 4 R^2 G^2 P_{in}\, n_{sp} \chi\, hf\, B = 3.38\times10^{-8}$

The optical noise power gets attenuated by the fiber propagation

The equivalent optical noise power

$(R\, P_{ni}) = I_{ni}$

$(R P_{ni})^2 = I_{ni}^2$

$P_{nf} = P_n\, \alpha\, \frac{L}{2}$

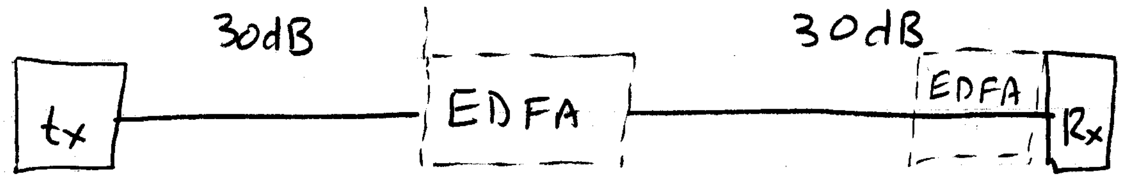
$I_{nf}^2 = (R\, P_{ni}\, \alpha\, \frac{L}{2})^2$

$= (R P_{ni})^2\, (\alpha \frac{L}{2})^2$

$= I_{ni}^2\, (\alpha\, \frac{L}{2})^2$

$= (3.38\times10^{-8})\, 10^{-6} = 3.38\times10^{-14}$

$SNR = \frac{6.4\times10^{-11}}{3.38\times10^{-14}} = 1893.5$



End

P_s	10mW		10nW	10μW
I_s	64μA			64pA
P_n	0			7.3μW
I_n	0			33.8pA
SNR	∞			1.89

middle

P_s	10mW	10μW	10mW	10μW
I_s	64μA		64μA	64pA
P_n	0		230μW	230nW
I_n	0		33.8nA	33.8×10^{-15}A
SNR	∞		1893	1893

What is the maximum transmission distance?

If you ignore thermal and shot noise

$$SNR = \frac{P_{in}}{4 n_{sp} \chi h f B}$$

$P_{in} = 52.8\ nW = -42.8\ dBm$

Power budget for 1st half of link

$PB = 52.8\ dB$

$L = 52.8/.2 = 264\ Km$

Total length $\boxed{L = 528\ Km}$

$(RNEP)^2 = 6.4 \times 10^{-27}$

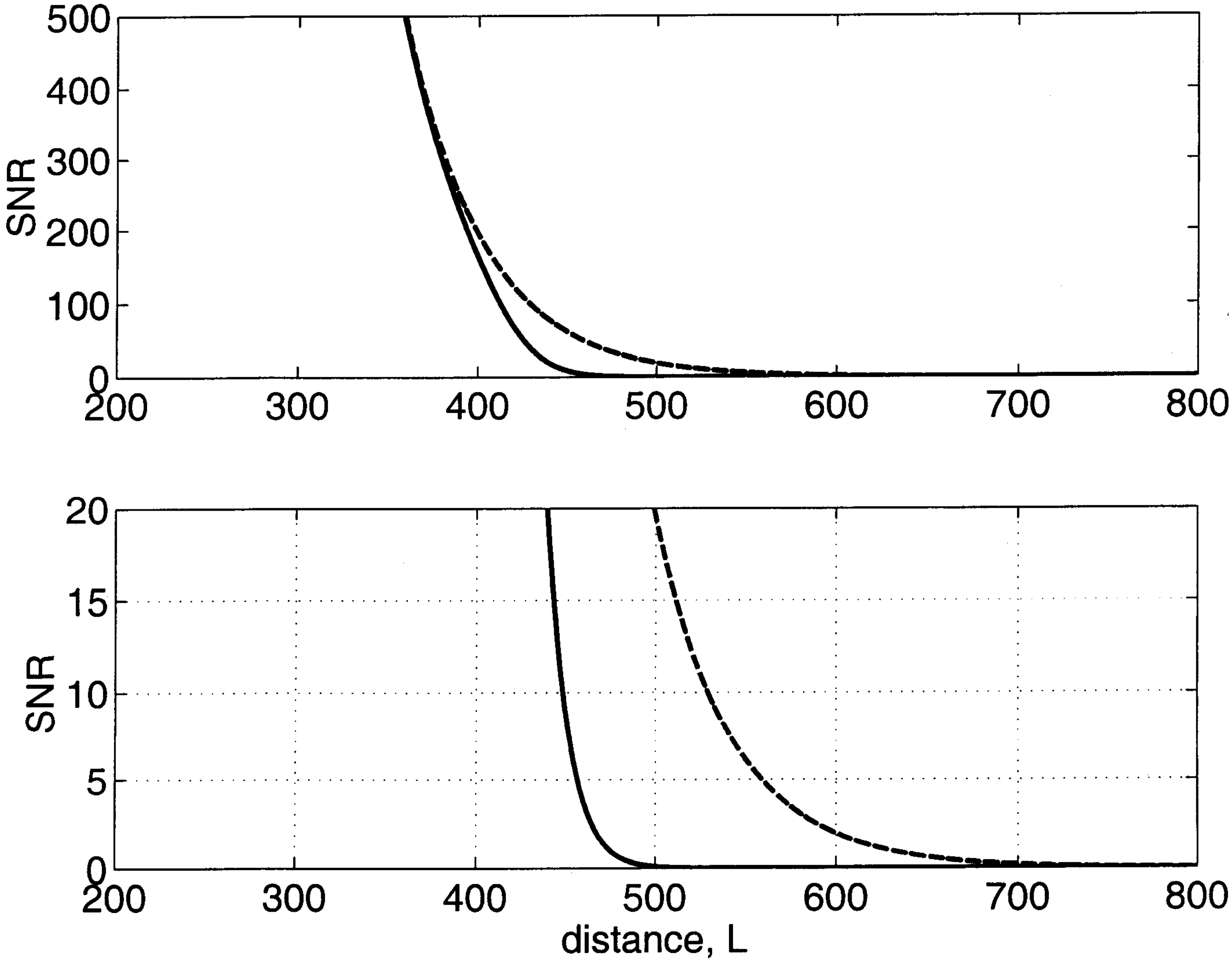
$4 R^2 G^2 n_{sp} \chi P_{in} \alpha = 4.88 \times 10^{-30} \not\gg (RNEP)^2$

You cannot ignore thermal noise

$$SNR = \frac{[R A P_t G]^2}{\left[(RNEP)^2 + 4 R^2 G^2 (A P_t) n_{sp} \chi h f \sqrt{A}\right] B}$$

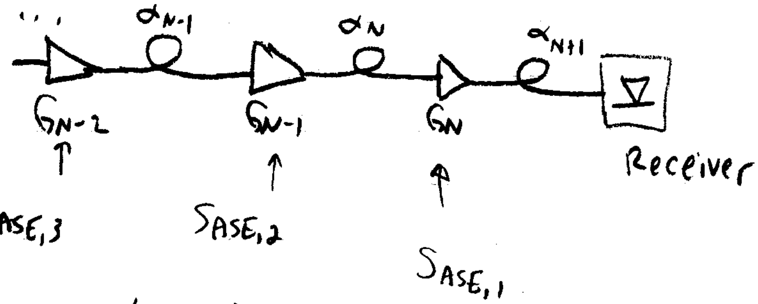
$A = 89.7\ dB$

$\boxed{L = 448.5\ Km}$



500
400
300
200
100
0
SNR
200
300
400
500
600
700
800
20
15
10
5
0
SNR
200
300
400
500
600
700
800
distance, L

If multiple amplifiers are needed



$$S_{ASE,1} = hf\, n_{sp}\, \chi\, (G_N - 1)\, \alpha_{N+1}$$

$$S_{ASE,2} = hf\, n_{sp}\, \chi \left[(G_{N-1} - 1)\, \alpha_N\right] (G_N)\, \alpha_{N+1}$$

$$S_{ASE,3} = hf\, n_{sp}\, \chi \left[(G_{N-2} - 1)\, \alpha_{N-1}\right] (G_{N-1})\, \alpha_N\, G_N\, \alpha_{N+1}$$

$$\vdots$$

$$S_{ASE,TOT} = hf\, n_{sp}\, \chi \left[\sum_{i=1}^{N-1} (G_{i-1} - 1) \prod_{j=i+1}^{N} G_j\, \alpha_j + G_N - 1\right] \alpha_{N+1}$$

$$\alpha = \prod_{i=1}^{N} \alpha_i \qquad G_{TOT} = \prod_{i=1}^{N} G_i$$

If the OA are uniformly spaced

$$\alpha_i = \alpha^{\frac{1}{N+1}}$$

If they all have the same gain

$$G_i = G$$

$$S_{ASE,TOT} = hf n_{sp} \chi \left[\sum_{i=1}^{N-1} (G-1) \prod_{j=i+1}^{N} G \alpha^{\frac{1}{N+1}} + (G-1) \right] \alpha^{\frac{1}{N+1}}$$

$$= hf n_{sp} \chi (G-1) \alpha^{1/N+1} \left[1 + \sum_{i=1}^{N-1} \prod_{j=i+1}^{N} G \alpha^{1/N+1} \right]$$

let $r = G \alpha^{1/N+1}$

$$S_{ASE\,TOT} = hf n_{sp} \chi (G-1) \alpha^{\frac{1}{N+1}} \left[1 + \underbrace{\sum_{i=1}^{N-1} \prod_{j=i+1}^{N} r}_{} \right]$$

$1 + r + r^2 + r^3 + \cdots r^{N-1}$

$$S_{ASE,TOT} = hf n_{sp} \chi (G-1) \alpha^{1/N+1} \frac{1-r^N}{1-r}$$

$G \gg 1$

$$S_{ASE,TOT} = hf n_{sp} \chi r \frac{1-r^N}{1-r}$$

$$= hf n_{sp} \chi G \alpha^{1/(N+1)} \frac{1 - G^N \alpha^{\frac{N}{N+1}}}{1 - G \alpha^{\frac{1}{N+1}}}$$

$$SNR^{(N)} = \frac{(R \alpha G^N P_{tx})^2}{\left[(R\,NEP)^2 + 4 R^2 \alpha G^N P_{tx} S_{ASE,TOT}^{(N)} \right] B}$$