Chapter 17  Optical Amplification

O-E  optical to electrical conversion
E-O  electrical to optical conversion

What determines the distance between regeneration?

Signal-to-Noise Ratio: SNR
Dispersion
Signal Power (attenuation)

What affects dispersion:
Fiber dispersion: Dispersion reduction
Dispersion compensation

Laser linewidth: DFB lasers

Modulation: Direct modulation of lasers
External modulators

Signal Power: Attenuation
Fiber component

OPTICAL AMPLIFIERS
Why optical amplifier vs. regeneration?

- Format independent 1Gbps 40Gbps
- Less complex

When will Optical Amplification not work?
- Dispersion dominates
- Too much noise
- Optical amplifier amplifies noise
- Optical amplifier generates noise

Criteria for Optical amplifier

1. High power gain
2. High external pumping efficiency
3. Small saturation effect
4. Large bandwidth
5. Low added noise
6. Small crosstalk
7. Proper operating wavelength
8. Low insertion loss

Common amplifiers: Semiconductor amplifier
Erbium doped fiber amplifier
Semiconductor Optical Amplifier

Laser cavity biased below threshold

\[ \text{Gain (dB)} = (1 - r_e^2) \left( 1 - r_i^2 \right) \frac{G_0}{1 - G_0 r_e r_i e^{-2BL/2}} \]

 Variation in gain with wavelength

\[ \text{Variation} = \left( 1 + \frac{G_0 r_e r_i}{1 - G_0 r_e r_i} \right)^2 \]

Example:

For a gain of 20 dB \((G_0 = 100)\),

what is the minimum reflectivity for a peak gain variation of 1 dB

\[ 10^{12} = 124 = \left( \frac{1 + 100 r_e r_i}{1 - 100 r_e r_i} \right)^2 \]

\[ \sqrt{124} \left( 1 - 100 r_e r_i \right) = 1 + 100 r_e r_i \]

\[ \frac{124 - 1}{100 + 124 (100)} = r_e r_i = 5.37 \times 10^{-4} \]

Very small
Figure 17.7  Methods to reduce the cavity end reflection: (a) anti-reflection coating, (b) cavity tilted at the Brewster angle, and (c) cavity facet tilted at the Brewster angle. In (b) and (c), note that the incident and output light is not parallel to the longitudinal direction of the cavity.
With multiple wavelength channels the carrier concentration varies randomly.

\[ E[\Delta g^2] \approx E[\Delta g_0^2] \quad \text{if} \quad T \gg \tau \]

\[ E[\Delta g_0^2] \left( \frac{1}{7} \right)^3 \left( \frac{1}{3} \right) \quad \text{if} \quad T \ll \tau \]

\[ \Delta g_0 = g(N) \frac{\Delta N_{ph}}{N_{ph, sat} + N_{ph, D}} \]

\[ \Delta N_{ph} : \text{one channel} \]

\[ N_{ph, 0} : \text{average} \]

For semiconductor amplifiers

\[ T \gg \tau \]
Semiconductor Optical Amplifiers (SOA)

- High gain ~ 20 dB
- High saturation power 5-10 dBm
- Large BW
- Can operate at any wavelength
- Compact
- Easy integration
- Integration into arrays

Polarization dependent

- High noise
- Cross talk between WDM channels

The crosstalk limits SOA in WDM applications
Erbium Doped Fiber Amplifier (EDFA)

Dope the core of an optical fiber with the rare earth ion Er$^{3+}$.

The energy diagram has a meta-stable state with an emission wavelength around $\lambda = 1530\text{ nm}$ and quick transitions into this state.

The population inversion is achieved by optically pumping at $\lambda = 980\text{nm}$ or $1480\text{nm}$.

The optical pumping is longitudinal rather than transverse resulting in better absorption of the optical pump.

The basic component of an EDFA are:
- Pump laser
- Isolator
- Coupler to combine signal and pump
- Erbium doped fiber
- Filter to eliminate pump light.
Figure 17.9 Energy levels of Er$^{3+}$. For each level, the ground state absorption (GSA) wavelength is the light wavelength needed to excite carriers from the ground state to the given level, and the excited state absorption (ESA) wavelength is the light wavelength needed to excite carriers from the given level to the metastable state $^4I_{13/2}$. 
With multiple wavelength channels the carrier concentration varies randomly.

\[ E[\Delta g^2] \approx E[\Delta g_0^2] \]

\[ E[\Delta g_0^2] \left( \frac{T}{7} \right)^3 \left( \frac{1}{3} \right) \quad T \gg \tau \]

\[ T < \tau \]

\[ \Delta g_0 = g(N) \frac{\Delta N_{ph}}{N_{ph,sat} + N_{ph,0}} \]

\[ \Delta N_{ph} : \text{one channel} \]

\[ N_{ph,0} : \text{average} \]

For semiconductor amplifiers

\[ T \gg \tau \]
980 or 1480 nm pump laser

isolator

weak input signal
coupler

EDF

filter

amplified output signal
The external pump rate is

\[ R_p = W_p N_i \]

where \( N_i \) is the ground state population

\[ W_p = \frac{\sigma_e P_p}{\hbar f_p A} \text{ s}^{-1} \]

\( \hbar f_p \) is the photon energy of the pump
\( A \) is the core area
\( \sigma_e \) is the absorption cross section
\( P_p \) is the pump power

The gain is given by

\[ g = \sigma_e (N_2 - N_i) \]

\( N_2 \) is the concentration in the metastable state
\( N_i \) is the concentration in the ground state
\( \sigma_e \) is the emission cross section

The stimulated emission rate becomes

\[ R_s = V_g \cdot g \cdot N_{ph} \]

\[ P_{in} = (V_g) \cdot (N_{ph} \cdot A \cdot \hbar f_s) \]

\[ R_s = \frac{\sigma_e P_{in}}{\hbar f_s A} (N_2 - N_i) \xrightarrow{\Delta} W_s (N_2 - N_i) \]

\[ W_s = \frac{\sigma_e P_{in}}{\hbar f_s A} \text{ s}^{-1} \]
Figure 17.13  Absorption cross section of Er$^{3+}$ at 800 nm.

Source: Reprinted, by permission, from Miniscalco, "Erbium-Doped Glasses for Fiber Amplifiers at 1500 nm," Figure 11 [10]. © 1991 by IEEE.

Figure 17.14  Absorption cross section of Er$^{3+}$ at 980 nm.

Source: Reprinted, by permission, from Miniscalco, "Erbium-Doped Glasses for Fiber Amplifiers at 1500 nm," Figure 12 [10]. © 1991 by IEEE.
**Figure 17.15** Absorption cross section of Er\(^{3+}\) at 1450 nm.

Source: Reprinted, by permission, from Miniscalco, "Erbium-Doped Glasses for Fiber Amplifiers at 1500 nm," Figure 7 [10]. © 1991 by IEEE.

**Figure 17.16** Emission cross section of Er\(^{3+}\) at 1540 nm.

Source: Reprinted, by permission, from Miniscalco. "Erbium-Doped Glasses for Fiber..."
The carrier rate equation is

\[
\frac{dN_2}{dt} = \frac{W_p}{\tau_{sp}} N_1 - \frac{W_s}{\tau_{sp}} (N_2 - N_1) - \frac{N_2}{\tau_{sp}} = -\frac{dN_1}{dt}
\]

pump net stimulated rate  
emission rate sponteneous recombination rate

\[\tau_{sp} \sim 10 \text{ ms}\]

at steady state

\[0 = \frac{W_p}{\tau_{sp}} N_1 - \frac{W_s}{\tau_{sp}} (N_2 - N_1) - \frac{N_2}{\tau_{sp}}\]

\[0 = 2W_p N_1 - 2W_s (N_2 - N_1) - \frac{2N_2}{\tau_{sp}}\]

\[0 = W_p (N_1 + N_2) - W_p (N_2 - N_1) - 2W_s (N_2 - N_1) - \frac{1}{\tau_{sp}} (N_2 - N_1) - \frac{1}{\tau_{sp}} (N_1 + N_2)\]

\[(N_2 - N_1) (W_p + 2W_s + \frac{1}{\tau_{sp}}) = (N_1 + N_2) (W_p - \frac{1}{\tau_{sp}})\]

\[N_2 - N_1 = \left(\frac{W_p - \frac{1}{\tau_{sp}}}{W_p + 2W_s + \frac{1}{\tau_{sp}}}\right) N_t\]

\[N_t = N_1 + N_2\] is the total carrier density
Figure 17.15  Absorption cross section of Er$^{3+}$ at 1450 nm.

Source: Reprinted, by permission, from Miniscalco, "Erbium-Doped Glasses for Fiber Amplifiers at 1500 nm," Figure 7 [10]. © 1991 by IEEE.

Figure 17.16  Emission cross section of Er$^{3+}$ at 1540 nm.

Source: Reprinted, by permission, from Miniscalco, "Erbium-Doped Glasses for Fiber Amplifiers at 1500 nm," Figure 8 [10]. © 1991 by IEEE.
With a high pumping rate

\[ W_p \gg W_s \quad W_p \gg \gamma_{Tsp} \]

\[ N_2 - N_1 = \frac{W_p}{W_p} N_t \]

\[ g = \sigma_e (N_2 - N_1) \approx \sigma_e N_t \frac{\Delta}{g^*} \]

\[ g = \sigma_e (N_2 - N_1) \]

\[ g = \sigma_e \left( \frac{W_p - \gamma_{Tsp}}{W_p + 2W_s + \gamma_{Tsp}} \right) N_t \quad N_{t} = \frac{g^*}{\sigma_e} \]

\[ g = g^* \left[ \frac{W_p - \gamma_{Tsp}}{W_p + 2W_s + \gamma_{Tsp}} \right] \]

Example:
Figure 17.15  Absorption cross section of Er$^{3+}$ at 1450 nm.

Source: Reprinted, by permission, from Miniscalco, "Erbium-Doped Glasses for Fiber Amplifiers at 1500 nm," Figure 7 [10]. © 1991 by IEEE.

Figure 17.16  Emission cross section of Er$^{3+}$ at 1540 nm.

Source: Reprinted, by permission, from Miniscalco, "Erbium-Doped Glasses for Fiber Amplifiers at 1500 nm," Figure 8 [10]. © 1991 by IEEE.
Pump at $\lambda_p = 14180\,\text{nm}$, $\sigma_{a} = 2.2 \times 10^{-31}\,\text{cm}^2$

Signal at $\lambda = 1522, 1530, 1538, 1546, 1554, 1562\,\text{nm}$, $L = 5\,\text{m}$

<table>
<thead>
<tr>
<th>$\lambda_f,(\text{nm})$</th>
<th>$\sigma_e\times10^{-21},\text{cm}^2$</th>
<th>$g^*,(\text{m}^{-3})$</th>
<th>$g,(\text{m}^{-1})$</th>
<th>$G,(\text{dB})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1522</td>
<td>4.28</td>
<td>3.42</td>
<td>1.77</td>
<td>3.8</td>
</tr>
<tr>
<td>1530</td>
<td>5.56</td>
<td>4.45</td>
<td>2.28</td>
<td>4.9</td>
</tr>
<tr>
<td>1538</td>
<td>5.22</td>
<td>4.18</td>
<td>2.14</td>
<td>4.6</td>
</tr>
<tr>
<td>1546</td>
<td>4.44</td>
<td>3.55</td>
<td>1.83</td>
<td>4.0</td>
</tr>
<tr>
<td>1554</td>
<td>4</td>
<td>3.20</td>
<td>1.65</td>
<td>3.6</td>
</tr>
<tr>
<td>1562</td>
<td>3.22</td>
<td>2.58</td>
<td>1.34</td>
<td>2.9</td>
</tr>
</tbody>
</table>

\[ g^* = \sigma_e n L \]

\[ g = g^* \frac{w_p - y_{Tp}}{w_p + 2w_s + y_{Tp}} \]

\[ G = (4.34g) L \]
**EDFA Gain Equalization**

- Gain equalization can be accomplished in several ways:
  a. Thin film filters
  b. Long period fiber gratings
  c. Chirped fiber Bragg gratings
The pump power is actually spatially dependent

\[
\frac{dP_p}{dz} = - \sigma_0 N_t P_p
\]

To relate \( N_t \) to the total carrier density \( N_t \), use Eq. 17.33

\[
N_2 - N_1 = \left( \frac{W_p - Y_{TsP}}{W_p + 2W_s + Y_{TsP}} \right) N_t
\]

\[
W_p (N_2 - N_1) + 2W_s (N_2 - N_1) + Y_{TsP} (N_2 - N_1) = W_p (N_2 + N_1) - Y_{TsP} (N_2 + N_1)
\]

\[
W_p (-2N_1) + 2W_s (N_2 - N_1) + Y_{TsP} (2N_2) = 0
\]

\[
-N_1 W_p + W_s (N_2 + N_1 - 2N_1) + Y_{TsP} (N_2 + N_1 - N_1) = 0
\]

\[
-N_1 (W_p + 2W_s + Y_{TsP}) + N_t (W_s + Y_{TsP}) = 0
\]

\[
N_1 = \left( \frac{W_s + Y_{TsP}}{W_p + 2W_s + Y_{TsP}} \right) N_t
\]

use this for pump power

\[
\frac{dP_p}{dz} = - \sigma_0 N_t P_p \frac{W_s + Y_{TsP}}{W_p + 2W_s + Y_{TsP}}
\]
The signal power is also spatially varying

\[ \frac{\text{Pin}}{\text{d}z} = g \text{Pin} \approx \text{Se} (N_2 - N_1) \text{Pin} \]

using Eq. 17.33

\[ \frac{\text{dPin}}{\text{d}z} = g^* \frac{W_p - \sqrt{\gamma_{sp}}}{W_p + 2W_s + \sqrt{\gamma_{sp}}} \quad \text{Pin} = \frac{g_0}{1 + W_s/W_{sat}} \text{Pin} \]

\[ g_0 = g^* \frac{W_p - \sqrt{\gamma_{sp}}}{W_p + \sqrt{\gamma_{sp}}} \]

\[ W_{sat} = \frac{1}{2} (W_p + \sqrt{\gamma_{sp}}) \]

\( W_p \) gets smaller along the propagation direction. Gain saturation effect is stronger at the output end of the amplifier.

The optimum amplifier length is a function of the pump power.
Figure 17.17  EDFA gain as a function of the amplifier length.

Source: Reprinted, by permission, from Giles and Desurvire, "Modeling Erbium-Doped Fiber Amplifiers," Figure 8b [12]. © 1991 by IEEE.
Erbium doped fiber amplifiers (EDFA)

- High power transfer from pump to signal (> 50%)  
- Saturation output: 10 - 25 dBm  
- Long gain constant (t > 10 ms) to overcome patterned effect and crosstalk  
- Polarization independent  
- Already in fiber domain

Large devices: m - Km  
Amplified spontaneous emission (ASE) noise