With a single mode waveguide then the intra-mode dispersion is of more concern

Intra-mode Dispersion has 2 parts
Material dispersion
Waveguide dispersion

The propagation delay is frequency dependent

$$\tau_g = \frac{1}{V_g} = \tau_g(\lambda_0) + (\lambda - \lambda_0)\frac{d\tau_g}{d\lambda} + \frac{1}{2}(\lambda - \lambda_0)^2 \frac{d^2\tau_g}{d\lambda^2} + \dots$$

$$D_{intra} \triangleq \frac{d}{d\lambda}\left(\frac{1}{V_g}\right) = \frac{d}{d\lambda}\left(\frac{d\beta_z}{d\omega}\right)$$

$$\Delta\tau_g \approx \Delta\lambda \, D_{intra} \, L$$

$$D_{intra} = \frac{d}{d\lambda}\left(\frac{d\beta_z}{d\omega}\right)$$
$$= \frac{d}{d\lambda}\left(\frac{d\beta_z}{d\beta_1}\frac{d\beta_1}{d\omega}\right)$$

$$D_{intra} \doteq \underbrace{\left(\frac{d\beta_z}{d\beta_1}\right)\left(\frac{d^2\beta_1}{d\lambda\, d\omega}\right)}_{\text{Material}} + \underbrace{\left(\frac{d^2\beta_z}{d\lambda\, d\beta_1}\right)\left(\frac{d\beta_1}{d\omega}\right)}_{\text{waveguide}}$$

Material Dispersion

$$D_{material} = \frac{d\beta_{zi}}{d\beta_1} \frac{d}{d\lambda}\left(\frac{d\beta_1}{d\omega}\right)$$

$$\beta_{zi}^2 = \beta_1^2 - K_i^2$$

$$\frac{n_2}{n_1} < \frac{\beta_{zi}}{\beta_1} < 1$$

for weakly guiding β_{zi} is close to β_1

$$\beta_{zi} = \sqrt{\beta_1^2 - K_i^2}$$

$$\frac{d\beta_{zi}}{d\beta_1} = \frac{1}{2}\left(\beta_1^2 - K_i^2\right)^{-1/2}\left(2\beta_1 - \cancel{2K_i}^{\,0} \frac{dK_i}{d\beta_1}\right)$$

$$\frac{d\beta_{zi}}{d\beta_1} = \frac{\beta_1}{\beta_{zi}}$$

Now for the second part

$$\beta_1 = n_1 \frac{\omega}{c}$$

$$\frac{d\beta_1}{d\omega} = \frac{1}{c}\underbrace{\left(n_1 + \omega \frac{dn_1}{d\omega}\right)}_{n_{1g}}$$

$$n_{1g} = n_1 + \omega \frac{dn_1}{d\omega}$$

$$= n_1 - \lambda \frac{dn_1}{d\lambda}$$

$$\frac{d\beta_1}{d\omega} = \frac{n_{1g}}{c}$$

$$D_{material} = \frac{1}{c}\left(\frac{\beta_1}{\beta_{z_i}}\right) \frac{d}{d\lambda}(n_{1g})$$

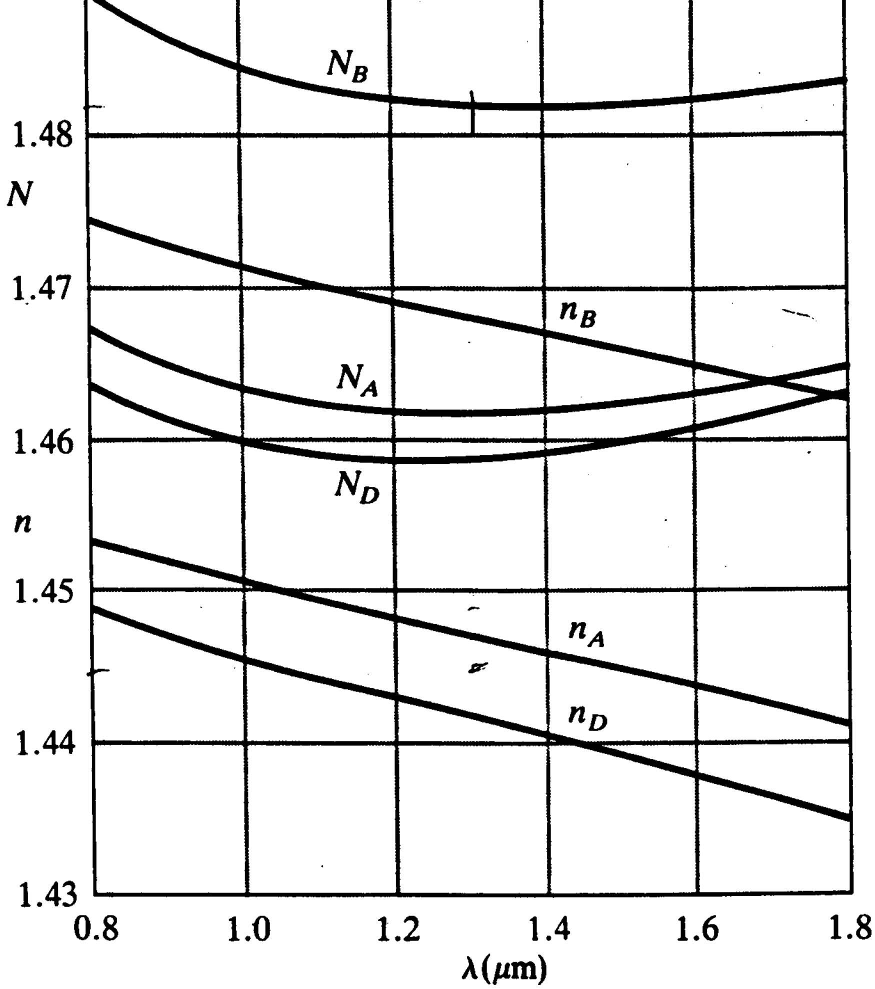

Figure 3.4.2 *The index of refraction, n, and group index,*

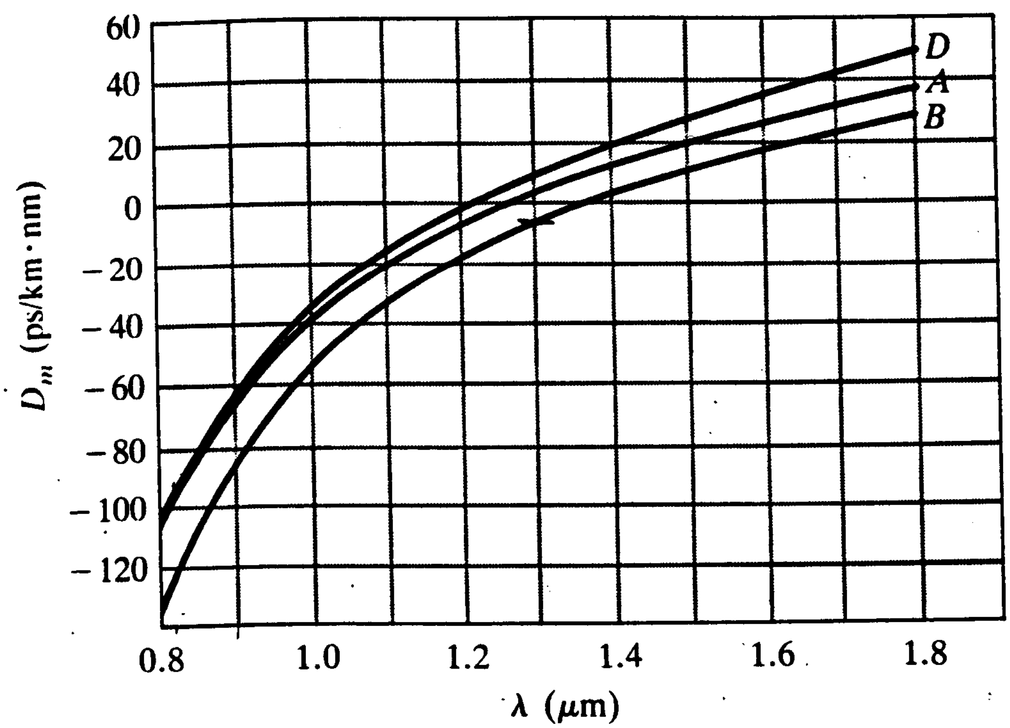

Figure 3.4.3 *Dispersion in optical fiber materials. These curves show dispersion for the three materials represented in Figure 3.4.2.*

Waveguide Dispersion

$$D_{waveguide} = \left(\frac{d^2 \beta_z}{d\lambda \, d\beta_1}\right)\left(\frac{d\beta_1}{d\omega}\right)$$

$$D_{waveguide} = \left(-\frac{n_{1g}}{c}\right)\left(\frac{n_{1g}}{n_1 \lambda}\right)\left[V^2 \frac{d^2}{dV^2}\left(\frac{\beta_z}{\beta_1}\right) + 2V \frac{d}{dV}\left(\frac{\beta_z}{\beta_1}\right)\right]$$

This is complicated

For weakly guiding fibers the following approximation is commonly used

$$D_{waveguide} = -\frac{(n_{1g} - n_{2g})}{c\lambda} V \frac{d^2}{dV^2}(Vb)$$

over the range $1.57 < V < 2.4$

$$V \frac{d^2}{dV^2}(Vb) \approx \frac{1.984}{V^2}$$

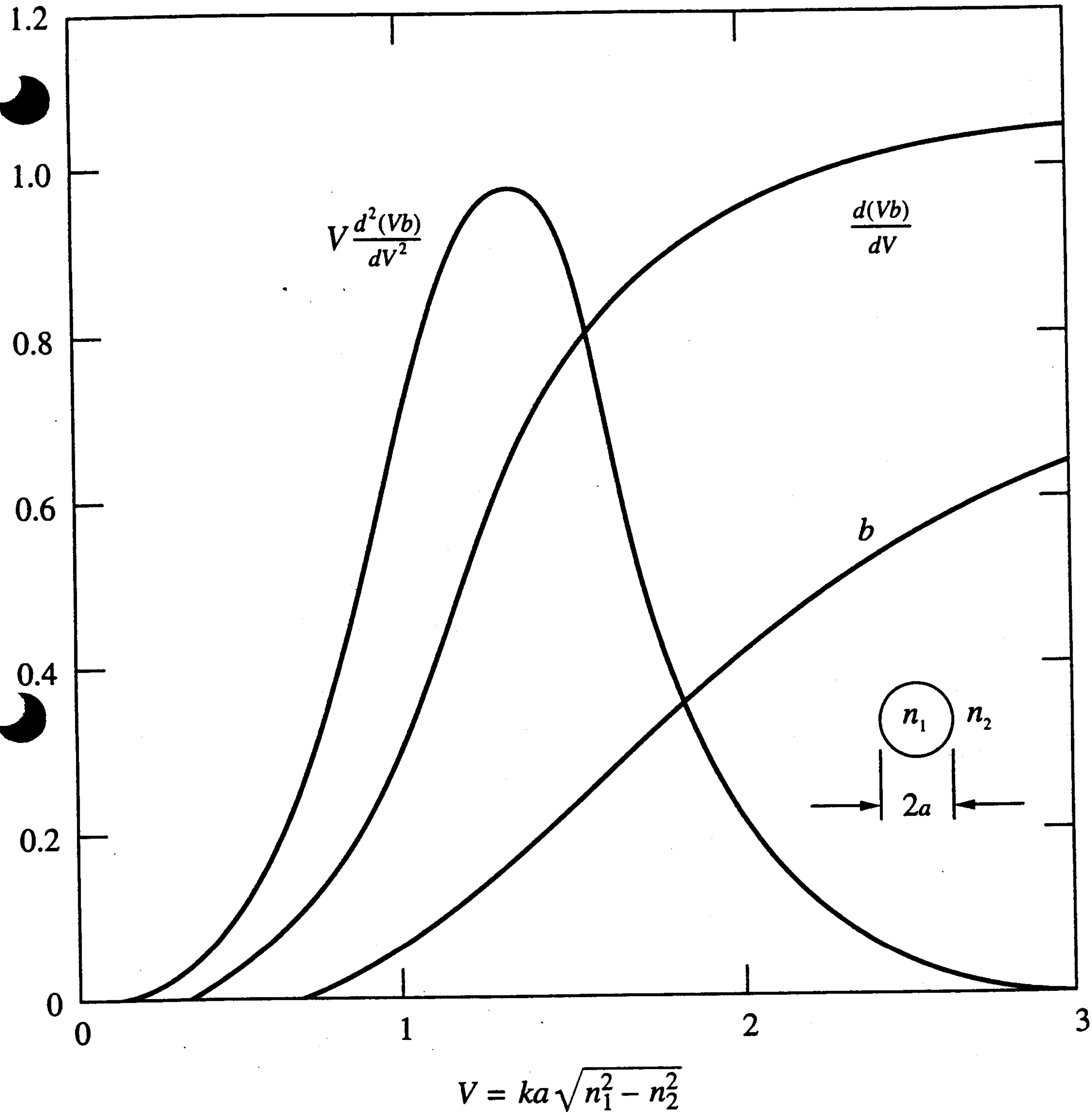

Figure 4.27 The factor used by Gloge to calculate waveguide dispersion.

The waveguide dispersion is negative while the material dispersion is positive in the wavelength region of interest.

The waveguide dispersion can be made to cancel the material dispersion at a particular wavelength.

$$D_{waveguide} \approx -\frac{(n_{1g}-n_{2g})}{c\lambda} \frac{1.984}{a^2 (2\pi)^2} \frac{\lambda^2}{(n_1^2-n_2^2)}$$

$$= -\left(\frac{n_{1g}-n_{2g}}{c}\right) \frac{1.984}{a^2 (2\pi)^2} \frac{\lambda}{(n_1^2-n_2^2)}$$

Adjust a such that $D_{intra} = 0$ at wavelength of interest.

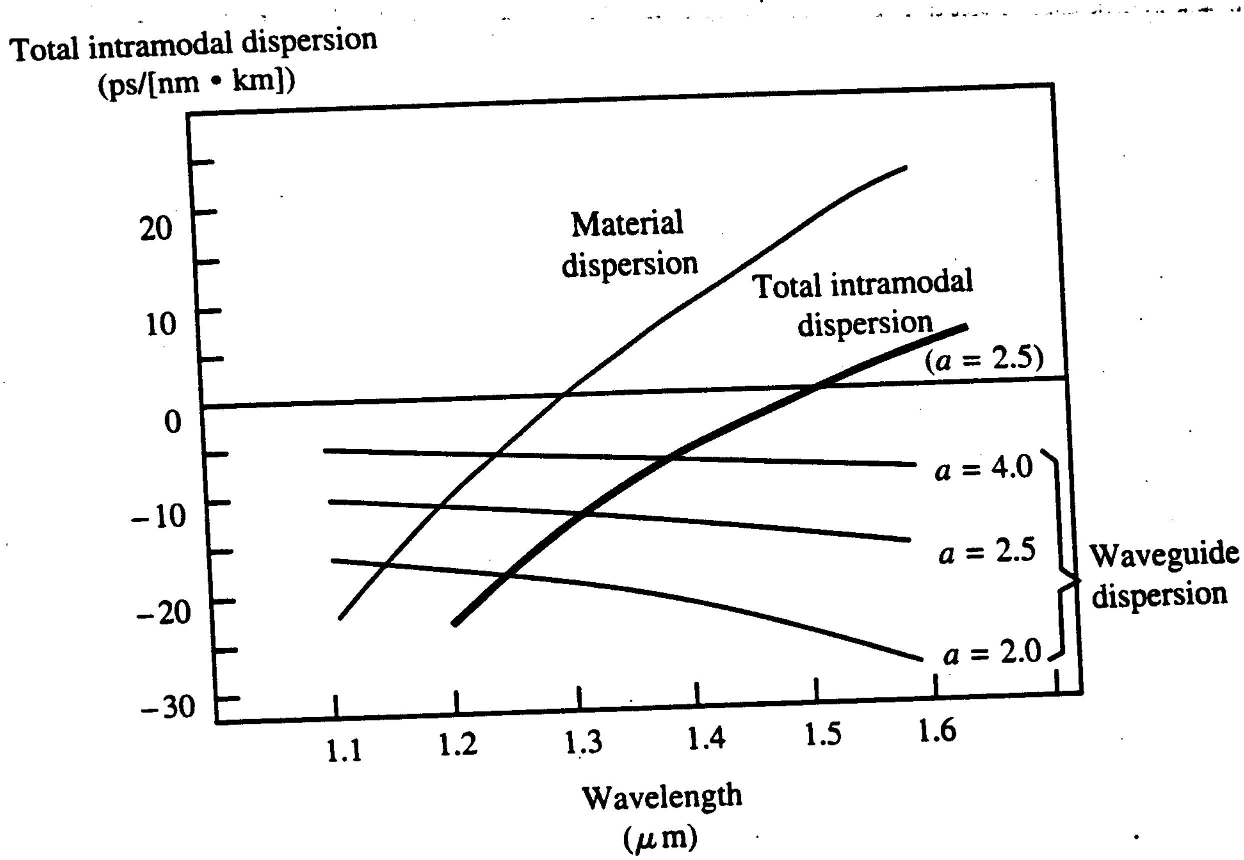

Figure 4.17 Total intramodal dispersion as a function of wavelength.

Now what is the total dispersion

D_{inter} is independent of wavelength

D_{inter} : ps/km

D_{intra} is dependent on wavelength linewidth

D_{intra} : $\frac{ps}{km\text{-}nm}$

$$D_{total}^2 = D_{inter}^2 + (D_{intra}\,\Delta\lambda)^2$$

Dispersion summary

$$D_{total}^2 = D_{inter}^2 + (D_{intra}\, \Delta\lambda)^2$$

$$D_{inter} = \frac{n_{2g}}{c} \Delta \, (b_1 - b_2)$$

$$= \frac{n_{2g}}{c} \Delta$$

$$D_{intra} = D_{material} + D_{waveguide}$$

$$D_{material} = \frac{1}{c} \left(\frac{\beta_1}{\beta_{zi}} \right) \frac{d}{d\lambda} (n_{1g})$$

$$D_{waveguide} = -\frac{(n_{1g} - n_{2g})}{c\lambda} V \frac{d^2}{dV^2} (Vb)$$

$$\approx -\frac{(n_{1g} - n_{2g})}{c\lambda} \left(\frac{1.984}{V^2} \right)$$