With a single mode waveguide then the intra-mode dispersion is of more concern.

Intra-mode Dispersion has 2 parts:
- Material dispersion
- Waveguide dispersion

The propagation delay is frequency dependent

$$\tau_g = \frac{1}{V_g} = \tau_g(\gamma_0) + (\gamma - \gamma_0) \frac{dT_g}{d\gamma} + \frac{1}{2} (\gamma - \gamma_0)^2 \frac{d^2T_g}{d\gamma^2} + \ldots$$

$$D_{\text{intra}} \triangleq \frac{d}{d\gamma} \left( \frac{1}{V_g} \right) = \frac{d}{d\gamma} \left( \frac{dB_2}{d\omega} \right)$$

$$\Delta \tau_g \approx \Delta \gamma \text{ D}_{\text{intra}} L$$

$$D_{\text{intra}} = \frac{d}{d\gamma} \left( \frac{dB_2}{d\omega} \right)$$

$$= \frac{d}{d\gamma} \left( \frac{dB_2}{dB_1} \frac{dB_1}{d\omega} \right)$$

$$D_{\text{intra}} = \left( \frac{dB_2}{dB_1} \right) \left( \frac{d^2B_1}{d\gamma d\omega} \right) + \left( \frac{d^2B_2}{d\gamma dB_1} \right) \left( \frac{dB_1}{d\omega} \right)$$

**Material**  
**Waveguide**
Material Dispersion

$$D_{\text{material}} = \frac{d\beta_{zi}}{d\beta_1} \left( \frac{d\beta_1}{dw} \right)$$

$$\beta_{zi}^2 = \beta_1^2 - k_i^2$$

$$\frac{n_2}{n_1} < \frac{\beta_{zi}}{\beta_1} < 1$$

for weakly guiding $\beta_{zi}$ is close to $\beta_1$

$$\beta_{zi} = \sqrt{\beta_1^2 - k_i^2}$$

$$\frac{d\beta_{zi}}{d\beta_1} = \frac{1}{2} \left( \beta_1^2 - k_i^2 \right)^{-\frac{1}{2}} \left( 2\beta_1 - 2k_i \frac{dk_i}{d\beta_1} \right)$$

$$\frac{d\beta_{zi}}{d\beta_1} = \frac{\beta_1}{\beta_{zi}}$$
Now for the second part

\[ \beta_i = n_1 \frac{\omega}{c} \]

\[ \frac{d\beta_i}{d\omega} = \frac{1}{c} \left( n_1 + \omega \frac{dn_1}{d\omega} \right) \]

\[ n_{ig} = n_1 + \omega \frac{dn_1}{d\omega} \]

\[ = n_1 - \gamma \frac{dn_1}{d\gamma} \]

\[ \frac{d\beta_i}{d\omega} = \frac{n_{ig}}{c} \]

\[ D_{\text{material}} = \frac{1}{c} \left( \frac{\beta_i}{\beta_{z_i}} \right) \frac{d}{d\gamma} (n_{ig}) \]
**Figure 3.4.2** The index of refraction, $n$, and group index, $N$.

**Figure 3.4.3** Dispersion in optical fiber materials. These curves show dispersion for the three materials represented in Figure 3.4.2.
Waveguide Dispersion

\[ D_{\text{waveguide}} = \left( \frac{d^2 \beta^2}{d\lambda d\beta} \right) \left( \frac{d\beta_1}{dw} \right) \]

\[ D_{\text{waveguide}} = -\left( \frac{n_{1g}}{c} \right) \left( \frac{n_{1g}}{n_{i1}} \lambda \right) \left[ V^2 \frac{d^2}{dV^2} \left( \beta^2 \right) + 2V \frac{d}{dV} \left( \beta^2 \right) \right] \]

This is complicated.

For weakly guiding fibers the following approximation is commonly used:

\[ D_{\text{waveguide}} = -\left( \frac{n_{1g} - n_{2g}}{c\lambda} \right) V \frac{d^2}{dV^2} (Vb) \]

over the range \( 1.58 < V < 2.4 \)

\[ V \frac{d^2}{dV^2} (Vb) \approx \frac{1.984}{V^2} \]
The factor used by Gloge to calculate waveguide dispersion.

\[ V = k a \sqrt{n_1^2 - n_2^2} \]
The waveguide dispersion is negative while the material dispersion is positive in the wavelength region of interest.

The waveguide dispersion can be made to cancel the material dispersion at a particular wavelength.

\[
D_{\text{waveguide}} = - \frac{(n_{1g} - n_{2g})}{c} \frac{1.984}{a^2 (2\pi)^3 (n_1^2 - n_2^2)} \lambda^2
\]

\[
= - \frac{(n_{1g} - n_{2g})}{c} \frac{1.984}{a^2 (2\pi)^3 (n_1^2 - n_2^2)} \lambda
\]

Adjust \( a \) such that \( D_{\text{int}} = 0 \) at wavelength of interest.
Total intramodal dispersion
(ps/[nm · km])

Material dispersion

Total intramodal dispersion
\(a = 2.5\)

\(a = 4.0\)

\(a = 2.5\)

\(a = 2.0\)

Waveguide dispersion

Wavelength
(\(\mu\)m)

1.1 1.2 1.3 1.4 1.5 1.6

-30 -20 -10 0 10 20

Figure 4.17 Total intramodal dispersion as a function of wavelength.
Now what is the total dispersion

Dinter is independent of wavelength

Dinter : \( \frac{ps}{km} \)

Dintra is dependent on wavelength linewidth

Dintra : \( \frac{ps}{km \cdot nm} \)

\[ D_{total}^2 = D_{inter}^2 + (D_{intra} \Delta \lambda)^2 \]
Dispersion Summary

\[ D_{\text{total}} = D_{\text{inter}}^2 + (D_{\text{intra}} \Delta x)^2 \]

\[ D_{\text{inter}} = \frac{n_{2g}}{c} \Delta (b_1 - b_2) \]

\[ = \frac{n_{2g}}{c} \Delta \]

\[ D_{\text{intra}} = D_{\text{material}} + D_{\text{waveguide}} \]

\[ D_{\text{material}} = \frac{1}{c} \left( \frac{\beta_1}{\beta_{2i}} \right) \frac{d}{dx} (n_{1g}) \]

\[ D_{\text{waveguide}} = -\left( \frac{n_{1g} - n_{2g}}{c} \right) V \frac{d^2}{dn^2} (Vb) \]

\[ \approx -\left( \frac{n_{1g} - n_{2g}}{c} \right) \left( \frac{1.984}{V^2} \right) \]