Intermodal Dispersion

$$\frac{\Delta\tau}{L} = \frac{1}{V_{g,min}} - \frac{1}{V_{gmax}}$$

This dispersion is independent of $\Delta\lambda$

Use the normalized propagation constant $b \equiv \frac{\beta^2 - k_2^2}{k_1^2 - k_2^2}$ $\quad 0<b<1$

$$\beta^2 = k_2^2 + b(k_1^2 - k_2^2)$$

$$\beta = k_0\left[n_2^2 + b(n_1^2 - n_2^2)\right]^{1/2}$$

$$= \frac{\omega n_2}{c}\left[1 + b\,\frac{n_1^2 - n_2^2}{n_2^2}\right]^{1/2}$$

$$\frac{n_1^2 - n_2^2}{n_2^2} = \frac{(n_1 - n_2)(n_1 + n_2)}{n_2^2} \qquad \text{if} \quad n_1 \approx n_2$$

$$\approx \frac{(n_1 - n_2)}{n_2}\,\frac{(2n_2)}{n_2}$$

$$= 2\Delta$$

$$\beta = \frac{\omega n_2}{c}\left[1 + 2b\Delta\right]^{1/2}$$

if $\Delta \ll 1$ $\qquad \sqrt{1 + 2b\Delta} \approx 1 + \Delta b$

$$\beta = \frac{\omega n_2}{c}(1 + \Delta b)$$

$$\frac{1}{V_g} = \frac{d\beta}{d\omega} = \frac{n_2}{c}(1 + \Delta b) + \frac{\omega}{c}(1 + \Delta b)\frac{dn_2}{d\omega}$$

$$= \frac{1}{c}(1 + \Delta b)\underbrace{\left(n_2 + \omega\frac{dn_2}{d\omega}\right)}_{n_{2g}}$$

$$n_{2g} = n_2 + \omega\frac{dn_2}{d\omega}$$

$$= n_2 - \lambda\frac{dn_2}{d\lambda}$$

$$\boxed{\frac{1}{V_g} = \frac{n_{2g}}{c}(1 + \Delta b)}$$

Dispersion $\quad \frac{\Delta\tau}{L} = \frac{1}{V_{g\,min}} - \frac{1}{V_{g\,max}}$

$$= \frac{n_{2g}}{c}\left[1 + \Delta b_1 - (1 + \Delta b_2)\right]$$

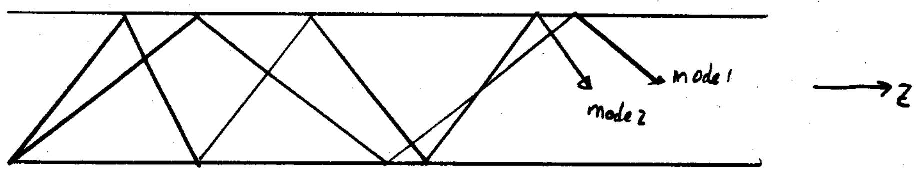
$$= \frac{n_{2g}}{c}\Delta(b_1 - b_2)$$

With lots of modes $\quad b_1 \simeq 1, \; b_2 \simeq 0$

$$\boxed{D = \frac{\Delta\tau}{L} = \frac{n_{2g}}{c}\Delta}$$

Intermodal dispersion for graded index optical fiber.

Let's start by looking qualitatively at the step index optical fiber. Look at propagation as a ray

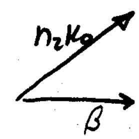


What is the pulse velocity?

The linear velocity is $\frac{c}{n_{2g}}$

The z-velocity has the ratio

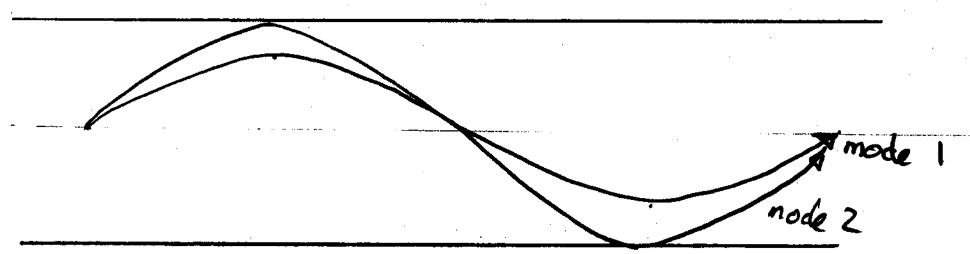
$$\frac{\beta}{n_2 k_0} = \frac{n_{eff}}{n_2}$$



So the z-velocity is $V_g = \left(\frac{c}{n_{2g}}\right)\frac{n_{eff}}{n_2}$

Different modes have different group velocity because the propagation distance is different.

Graded index optical fiber



Mode 2 has a larger propagation distance but since the speed is $\frac{c}{n}$ the velocity decreases the higher the mode goes.

So the dispersion is a little bit better for graded index optical fiber.

$$\boxed{D = \frac{n_{2g}\,\Delta^2}{8c}}$$

In class example

A standard single mode fiber (SMF28) is used with a laser that has a wavelength of $\lambda = 850nm$. What is the dispersion?

$a = 4.1 \mu m$

From Table 1.1

$n_2(633nm) = 1.457$

$n_2(1000nm) = 1.45$

$$n_2(850nm) = 1.457 + (850-633)\frac{(1.45-1.457)}{(1000-633)}$$ Interpolation

$n_2 = 1.4529$

$n_1 = n_2(1.0036) = 1.4581$

$$V = \left(\frac{2\pi}{0.85}\right)(4.1)\sqrt{(1.4581^2 - 1.4529^2)}$$

$$V = 3.73$$

Use Figure 3.12 from the book

There are 2 modes

$n_{eff1} = 1.4620$

$n_{eff2} = 1.4611$

but we need to convert these to normalized parameters to use with our specific case

$$b_1 = \frac{n_{eff}^2 - n_2^2}{n_1^2 - n_2^2} = \frac{1.4620^2 - 1.46^2}{1.4628^2 - 1.46^2} = 0.7141$$

$$b_2 = \frac{1.4611^2 - 1.46^2}{1.4628^2 - 1.46^2} = 0.3926$$

$$D = \frac{1}{V_{g\,min}} - \frac{1}{V_{g\,max}} = \frac{n_{2g}}{c}(1+\Delta b_1) - \frac{n_{2g}}{c}(1+\Delta b_2)$$

$$= \frac{n_{2g}}{c}\Delta(b_1 - b_2)$$

$$= \left(\frac{1.4677}{3\times10^8}\right)(0.0036)(0.7141 - 0.3926)$$

$$D = 5.66 \frac{ps}{m}$$

$$\boxed{D = 5.66 \frac{ns}{Km}}$$

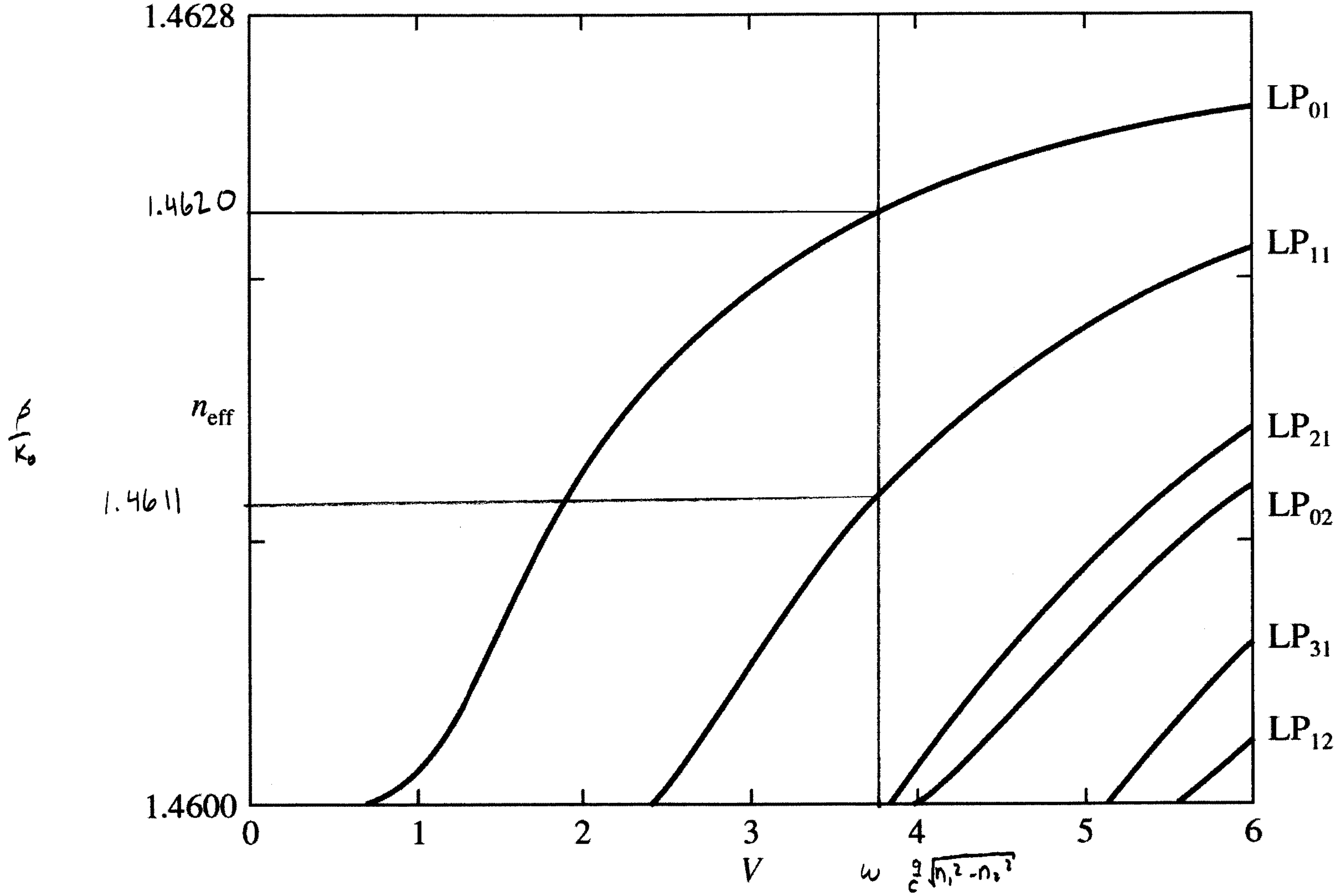


Figure 3.12 Normalized propagation constant n_{eff} as function of normalized frequency V for some of the guided modes of the optical fiber, $n_{eff} = \beta/k_0$. The fiber parameters are $n_1 = 1.4628$, $n_2 = 1.4600$, and $a = 4.7$ μm.

Dispersion of graded index optical fiber

$n_{2g} = 1.491$

$\Delta = 0.02$

$$D = \frac{n_{2g}\,\Delta^2}{8c} = \frac{(1.491)(0.02)^2}{(8)(3\times10^8)} = 0.25\ \frac{ps}{m}$$

$$D = 0.25\ \frac{ns}{km}$$

This is a lot better than the step index optical fiber