# EO modulator Configurations

1. Polarimetric
2. Mach-Zehnder (MZ)
3. Fabry Perot
4. Waveguide coupling

The most common configuration is the MZ modulator.

We will cover the polarimetric and MZ modulators.

The polarimetric uses the relative phase delay between polarizations

The MZ uses the absolute phase delay of a particular polarization

Lets look at GaAs

$$n_x = n + \tfrac{1}{2} n^3 r_{41} E_z$$
$$n_y = n - \tfrac{1}{2} n^3 r_{41} E_z$$
$$n_z = n$$

The relative phase delay is

$$\phi = (n_x - n_y) KL = n^3 r_{41} E_z KL$$

The absolute phase delay is

$$\phi = n_x KL = \tfrac{1}{2} n^3 r_{41} E_z KL$$

What about $LiNbO_3$

relative phase

$$\phi = (n_z - n_x)\, KL$$
$$= (n_o - n_E) - \tfrac{1}{2} E_z \left(n_o^3 r_{13} - n_E^3 r_{33}\right) KL$$

absolute phase

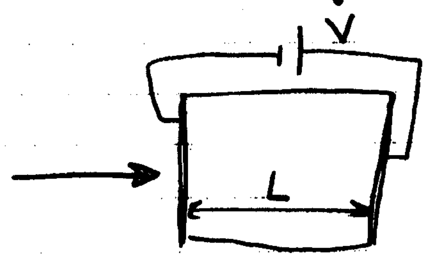
$$\phi = n_x KL = -\tfrac{1}{2}\, n_o^3\, r_{13}\, E_z\, KL$$

with GaAs relative phase was double absolute phase

with $LiNbO_3$ absolute phase was larger

There are also two basic modulator configurations

Longitudinal : Applied field in direction of signal propagation

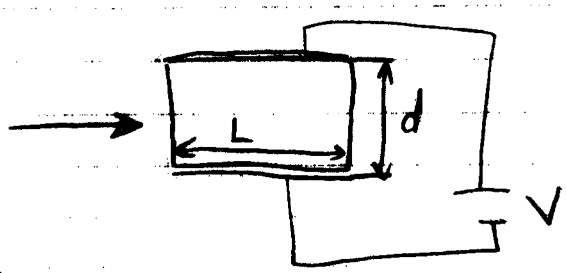



$LiNbO_3$ absolute phase

$$\phi = \frac{1}{2} n_o^3 r_{13} \frac{V}{L} k L$$

$$\phi = \frac{1}{2} n_o^3 r_{13} V k$$

Transverse : Applied field perpendicular to direction of propagation



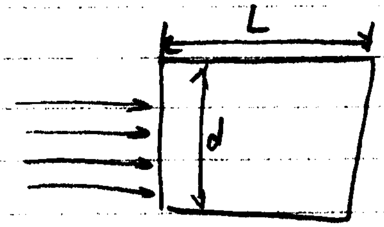

$LiNbO_3$ absolute phase

$$\phi = \frac{1}{2} n_o^3 r_{13} \frac{V}{d} k L$$

$$= \frac{1}{2} n_o^3 r_{13} V k \frac{L}{d}$$

Bulk transverse modulator

beam width ~ 1mm

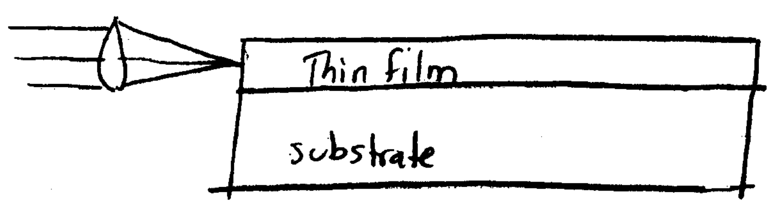



$d = 1\,mm$

$\frac{L}{d} = 1000 \Rightarrow L = 1m$

This is too big
Difficult to make a 1m long 1mm thick slab.

Integrated modulator:
Make the modulator into a waveguide




$d = 0.5 \mu m$

$L = (1000)(0.5 \mu m) = 500 \mu m$ much more realistic

Let's do a comparison

$LiNbO_3$ : $r_{13} = 9.6 \times 10^{-12}$ m/V

$n_o = 2.286$

$\lambda = 0.633$

Longitudinal

$$\pi = \phi = (1/2)(n_o^3)(r_{13})\, V \left(\frac{2\pi}{\lambda}\right)$$

$$\pi = (1/2)(2.286^3)(9.6 \times 10^{-12})\left(\frac{2\pi}{.633 \times 10^{-6}}\right) V$$

$$V = \frac{(.633 \times 10^{-6})}{(2.286^3)(9.6 \times 10^{-12})} = 5520 \text{ Volts}$$

can't really change voltage

Transverse

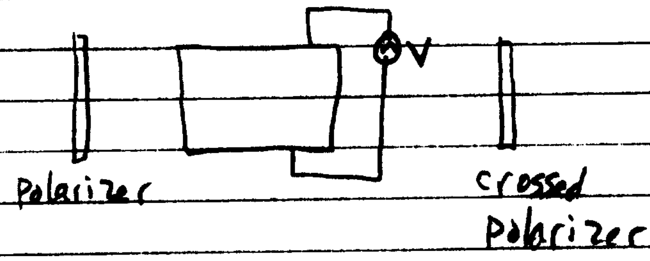
$$\pi = 1/2\; n_o^3\; r_{13}\; \frac{2\pi}{\lambda}\; \frac{L}{d}\; V$$

$$V = 5520\, \frac{d}{L}$$

increase $\frac{L}{d}$ to reduce voltage

if $\frac{L}{d} = 1000$ $V = 5.5$ Volts

Polarimetric Modulator (z-cut $LiNbO_3$)




retardation between polarizations

$$\Gamma = \left[(n_o - n_E) - \tfrac{1}{2} \frac{V}{d} (n_o^3 r_{13} - n_E^3 r_{33})\right] \frac{2\pi}{\lambda} L$$

polarization directions must be at $\pm 45°$

Assume built in retardation is $m 2\pi$

$$(n_o - n_E) \frac{2\pi}{\lambda} L = m\, 2\pi$$

$$\Gamma = \frac{\pi}{\lambda}(n_o^3 r_{13} - n_E^3 r_{33}) \frac{L}{d} V$$

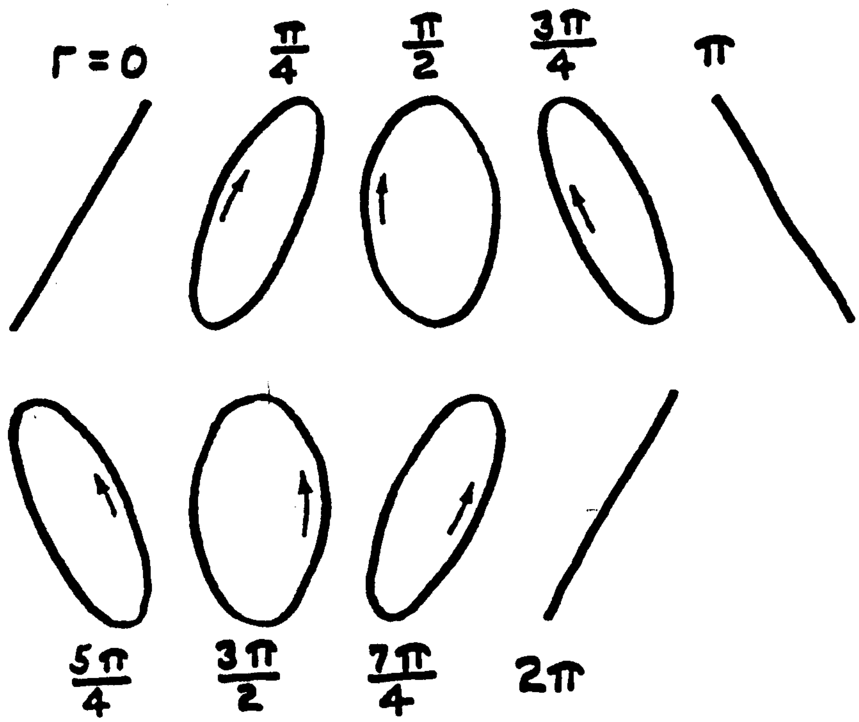
$$\frac{I_{out}}{I_{in}} = \sin^2\left(\frac{\Gamma}{2}\right) = I_o \sin^2\left[\frac{\pi}{\lambda} \frac{1}{2} (n_o^3 r_{13} - n_E^3 r_{33}) \frac{L}{d} V\right]$$

$$I = I_o \sin^2\left[\pi \frac{V}{V_\pi}\right]$$

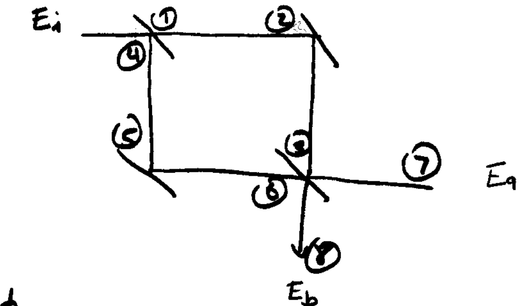
$$V_\pi = \lambda (2) \left(\frac{d}{L}\right) \frac{1}{n_o^3 r_{13} - n_E^3 r_{33}}$$

# Polarization Modulation

Addition of two coherent waves polarized at right angles



# Mach - Zehnder Interferometer (MZI)




① $t_1 E_i$

② $t_1 E_i e^{-j\phi_1}$ path 1

③ $t_1 E_i e^{-j(\phi_1+\phi_2)}$

④ $r_1 E_i$

⑤ $r_1 E_i e^{-j\phi_3}$ path 2

⑥ $r_1 E_i e^{-j(\phi_3+\phi_4)}$

⑦ $t_1 r_2 E_i e^{-j\overbrace{(\phi_1+\phi_2)}^{\phi_a}} + r_1 t_2 E_i e^{-j\overbrace{(\phi_3+\phi_4)}^{\phi_b}}$

if $t_1 r_2 = r_1 t_2$

$$E_a = (rtE_i)\left(e^{-j\phi_a} + e^{-j\phi_b}\right)$$

$$= (rtE_i)(2)\, e^{-j\frac{(\phi_a+\phi_b)}{2}} \left( e^{-j\left(\frac{\phi_a-\phi_b}{2}\right)} + e^{+j\left(\frac{\phi_a-\phi_b}{2}\right)} \right)$$

$$E_a = \left(rtE_i\, 2\, e^{-j\bar{\phi}}\right) \cos\left(\frac{\Delta\phi}{2}\right)$$

assume $r_1 = r_2 = \frac{1}{\sqrt{2}} e^{j\pi/4}$

$t_1 = t_2 = \frac{1}{\sqrt{2}} e^{-j\pi/4}$

$$I_a = |rt\,2|^2 |E_i|^2 \cos^2\left(\frac{\Delta\phi}{2}\right)$$

$$\boxed{I_a = \frac{I_0}{2}\left(1 + \cos\Delta\phi\right)}$$


⑧ $t_1 t_2 E_i e^{-j\phi_a} + r_1 r_2 E_i e^{-j\phi_b}$

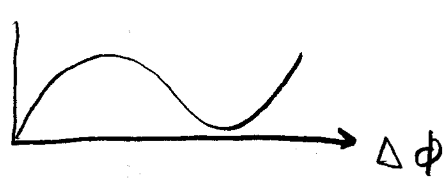
$t_1 t_2 = \frac{1}{2} e^{-j\pi/2}$

$r_1 r_2 = \frac{1}{2} e^{+j\pi/2}$

$$E_b = \frac{1}{2} e^{-j\pi/2} E_i \left( e^{-j\phi_a} - e^{-j\phi_b} \right)$$

$$= \frac{1}{2} e^{-j\pi/2} E_i\, e^{-\bar{\phi}} \left( e^{-j\frac{\Delta\phi}{2}} - e^{+j\frac{\Delta\phi}{2}} \right)$$

$$I_b = I_0 \sin^2\frac{\Delta\phi}{2} = \frac{I_0}{2}\left(1 - \cos\Delta\phi\right)$$



$\Delta\phi = \frac{2\pi}{\lambda} L \, \Delta n$ $\Delta n$ is for single polarization

Typically it is operated in a push-pull arrangement with a +V applied to one arm and a -V applied to the other arm

$\phi_1 = \frac{2\pi}{\lambda} L \Delta n$ $\phi_2 = -\frac{2\pi}{\lambda} L \Delta n$

$\Delta\phi = \phi_1 - \phi_2 = \frac{\pi}{\lambda} L \, \Delta n$

$\Delta\phi = \frac{\pi}{\lambda} L \left(\frac{n_E^3}{2}\right) r_{33} \frac{V}{d}$

$\Delta\phi = \frac{\pi}{\lambda}\left(\frac{L}{d}\right)\left(\frac{n_E^3}{2}\right) r_{33} \; V$

$$\frac{I_{out}}{I_{in}} = \cos^2\left[\left(\frac{\pi}{2\lambda}\right)\left(\frac{L}{d}\right)\left(\frac{n_E^3}{2}\right) r_{33} \, V\right]$$

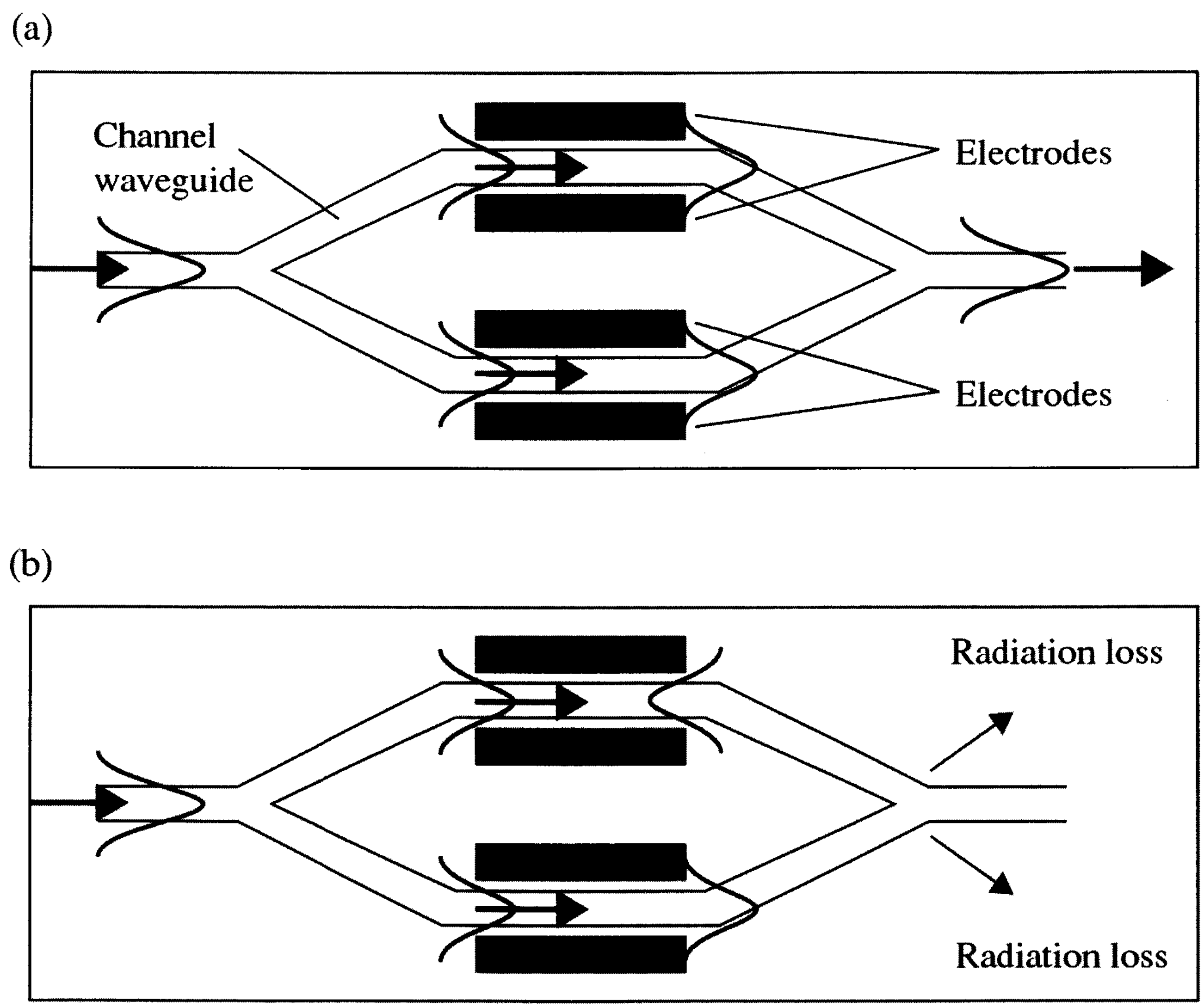



**Figure 9.10** Schematic drawing of the top view of a Mach–Zehnder interferometer modulator. (a) Recombination with in-phase beams. (b) Recombination with out-of-phase beams.

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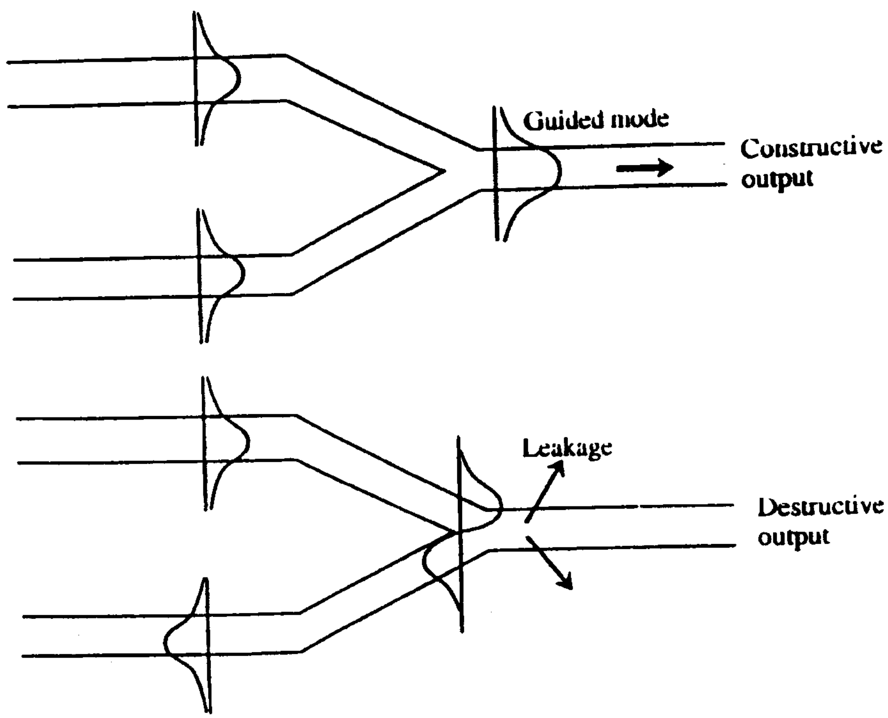
# Effect of 180° Phase Shift Between Signals Entering Y-Junction

S.L. Chuang, Physics of Optoelectronic Devices




$$P_{out} = \frac{1}{4}\left|e^{i\beta_a \ell} + e^{i\beta_b \ell}\right|^2$$

$$= \cos^2\left(\frac{\Delta\beta}{2}\ell\right)$$

Scientific Atlanta

Comparison between MZI and polarization filtering

**Polarization**

$$\frac{I_{out}}{I_{in}} = \cos^2\left(\frac{\Gamma}{2}\right)$$

$$= \frac{1}{2}(1+\cos\Gamma)$$

$$\Gamma = \frac{2\pi}{\lambda} L\,(n_1 - n_2)$$

**MZI**

$$\frac{I_{out}}{I_{n}} = \cos^2(\Delta\phi)$$

$$= \frac{1}{2}(1+\cos\Delta\phi)$$

$$\Delta\phi = \frac{2\pi}{\lambda} L\,\Delta n_1$$

**$LiNbO_3$**

$$\Gamma = \frac{2\pi}{\lambda}\frac{L}{d}\left(\frac{n_E^3}{2} r_{33} - \frac{n_o^3}{2} r_{13}\right) V$$

$$\Delta\phi = \frac{2\pi}{\lambda}\frac{L}{d}\left(\frac{n_E^3}{2} r_{33}\right) V$$

or

$$\frac{2\pi}{\lambda}\left(\frac{L}{d}\right)\left(\frac{n_o^3}{2} r_{13}\right) V$$

**GaAs**

$$\Gamma = \frac{2\pi}{\lambda}\frac{L}{d}\left(n^3 r_{41}\right) V$$

$$\Delta\phi = \frac{2\pi}{\lambda}\left(\frac{L}{d}\right)\left(\frac{n^3}{2} r_{41}\right) V$$

Requires mixed polarization

Requires single polarization

Frequency dependence

both the polarization filtering and MZ modulators have an average phase term that we ignored before

$$E = E_0 \cos\left(\frac{\Delta\phi}{2}\right) \cos(\beta z - \omega t - \bar{\phi})$$

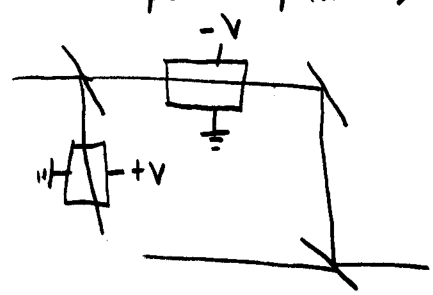
the frequency is $\omega = \frac{\partial \phi}{\partial t} = \frac{\partial}{\partial t}(\beta z - \omega t - \phi)$

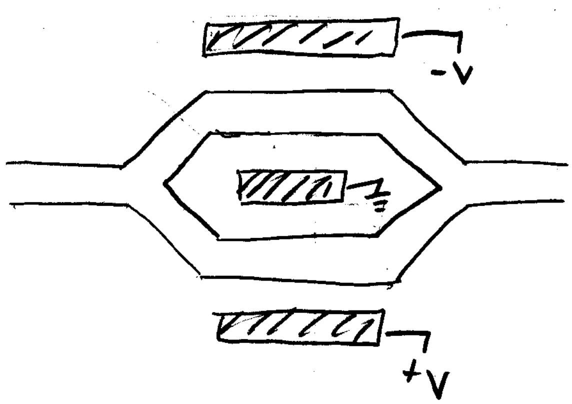
$$\omega = \omega + \underbrace{\frac{\partial \phi}{\partial t}}_{\text{frequency shift}}$$

So the modulator creates a frequency chirp

we can eliminate the chirp by making $\bar{\phi} = 0$.

This is done as a push-pull MZ




$$\phi_a = \frac{n_E^3}{2} r_{33} \left(\frac{L}{d}\right) V$$

$$\phi_b = -\frac{n_E^3}{2} r_{33} \left(\frac{L}{d}\right) V$$

$$\phi_a + \phi_b = 0$$

$$\phi_a - \phi_b = n_E^3 r_{33} \left(\frac{L}{d}\right) V$$

$\boxed{\bar{\Phi} = 0} \rightarrow$ No frequency chirp

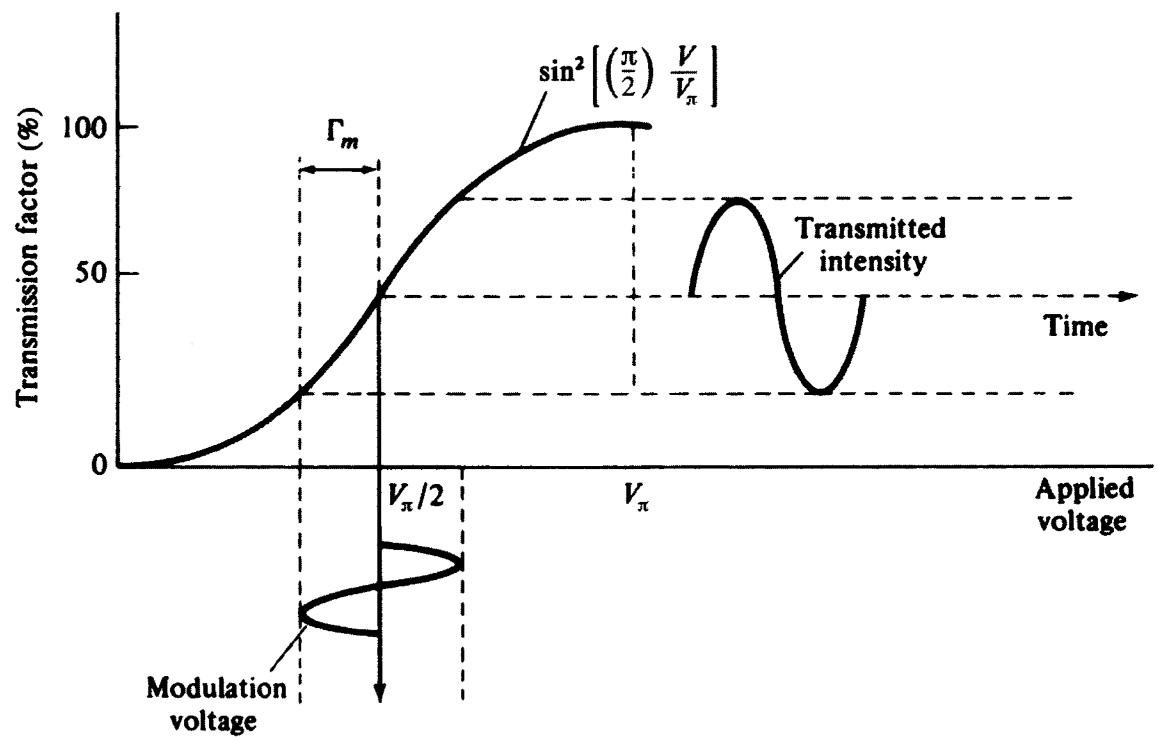



**Figure 9.5** Transmission factor of a cross-polarized electro-optic modulator as a function of an applie voltage. The modulator is biased to the point $\Gamma_B = \pi/2$, which results in a 50% intensity transmission. A small applied sinusoidal voltage modulates the transmitted intensity about the bias point.

# 4. Waveguide Coupling

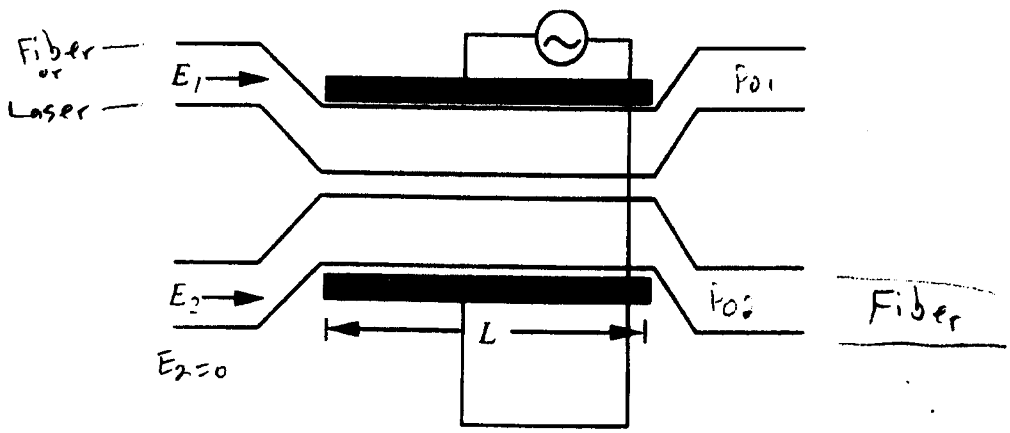



**Figure 14.10** EO amplitude modulator using waveguide coupling.

Power is coupled between waveguides

$$\mathcal{E}(x,y,z,t) = E_1(z)\, E_{T,1}(x,y)\, e^{j(\omega t - \beta_1 z)} + E_2(z)\, E_{T,2}\, e^{j(\omega t - \beta_2 z)}$$

The coupled mode equations are

$$\frac{dE_1}{dz} = -jKE_2\, e^{j2\delta z} \qquad K \text{ is coupling constant}$$

$$\frac{dE_2}{dz} = -jKE_1\, e^{-j2\delta z} \qquad \delta \text{ is mismatch factor}$$

$P_{in}$ is input power in waveguide 1

$$P_{01} = P_{in} - P_{02}$$

$$P_{02} = P_{in}\frac{K^2}{K^2+\delta^2}\sin^2\left[\left(K^2+\delta^2\right)^{1/2} z\right]$$

$$\delta \approx \frac{1}{2}(\beta_1 - \beta_2) = \frac{2\pi}{\lambda}\Delta n$$

with zero applied voltage

$$P_{o2} = P_{in}\frac{\kappa^2}{\kappa^2}\sin^2\left[\kappa L\right]$$

for 100% crossover $\qquad P_{o2} = P_{in}$

$$\kappa L = \pi/2$$

$$L_\pi = \frac{\pi}{2\kappa}$$

$$P_{o2} = P_{in}\frac{\kappa^2}{\kappa^2+\delta^2}\sin^2\left[\sqrt{\kappa^2+\delta^2}\,\frac{\pi}{2\kappa}\right]$$

for extinction $\qquad \delta = \sqrt{3}\,\kappa$

$$P_{o2} = P_{in}\frac{\kappa^2}{4\kappa^2}\sin^2\left[2\kappa\,\frac{\pi}{2\kappa}\right] = 0$$

If $L \neq \frac{\pi}{2\kappa}$ then 100% crossover is not possible

$$P_{o2} = P_{in}\frac{\kappa^2}{\kappa^2+\delta^2} < P_{in}$$

This results in insersion loss or lower modulation depth

$$P_{o1}(L) = P_{in}\left(1 - \frac{2\kappa^2}{\kappa^2+\delta^2}\sin^2[(\kappa^2+\delta^2)^{1/2}L/2]\right)^2 \qquad \textbf{[14.95]}$$

$P_{o2} = P_{in} - P_{o1}$

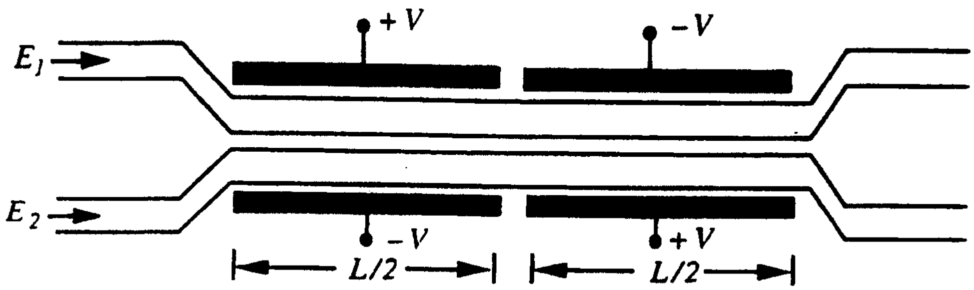



**Figure 14.11** Alternating $\Delta\beta$ waveguides.

Complete power transfer when

$$\frac{2\kappa^2}{\kappa^2+\delta^2}\sin^2\left[(\kappa^2+\delta^2)^{1/2}\frac{L}{2}\right] = 1$$

So $\delta$ can be adjusted for complete power transfer. for any $L$

Requires longer length device than other modulators

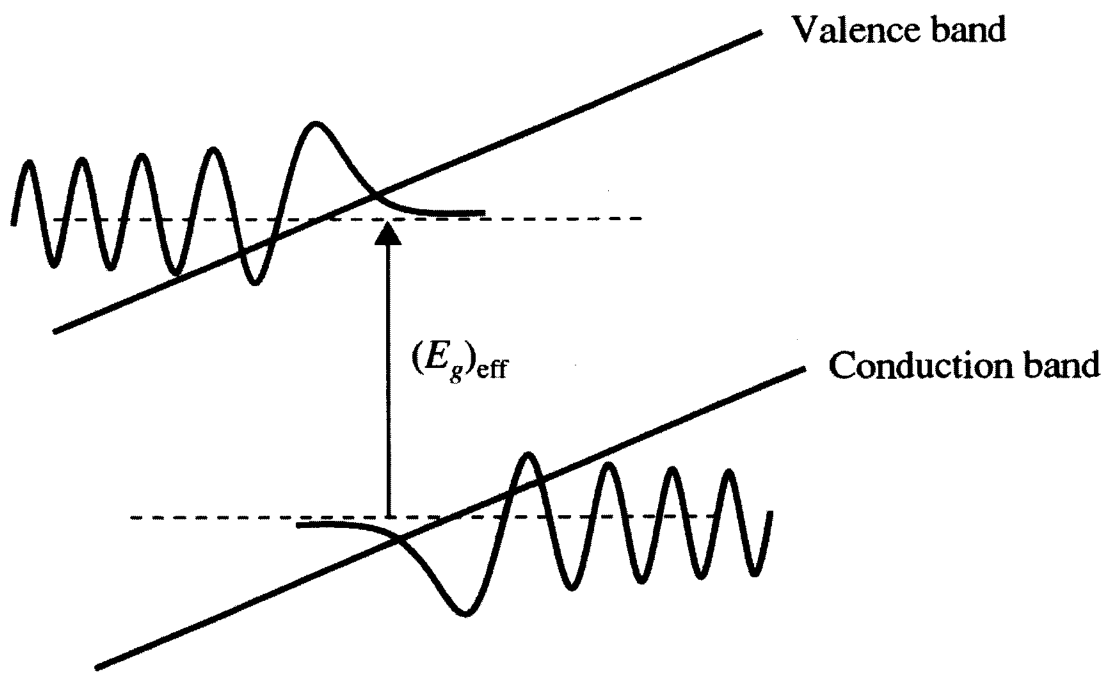



**Figure 9.14** Schematic drawing of the Franz–Keldysh effect (FKE) in semiconductors. The wavefunctions of carriers become Airy functions that tunnel into the forbidden gap.

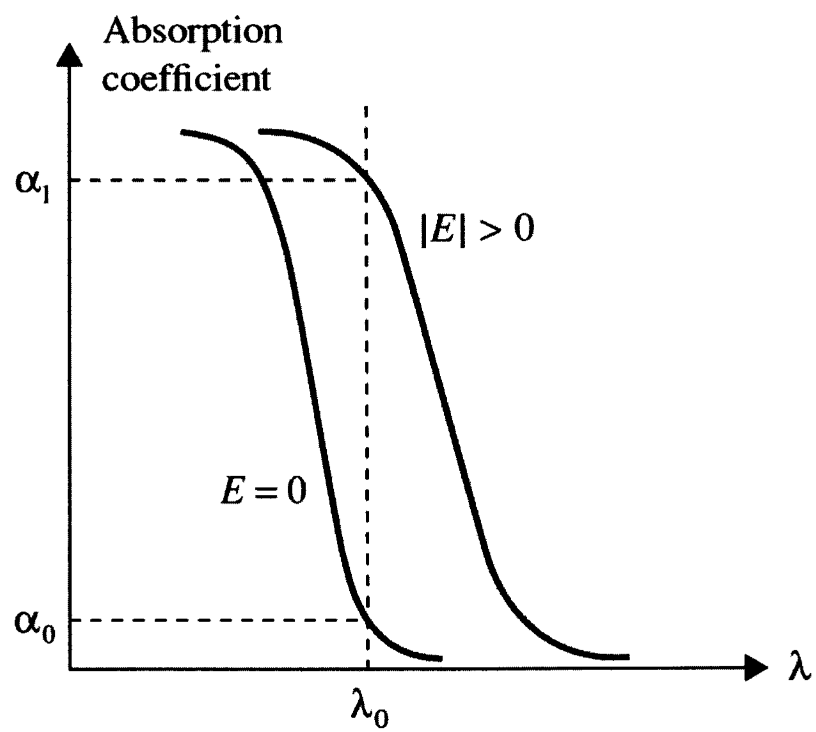



**Figure 9.15** The Franz–Keldysh effect in semiconductors leads to a shift (red shift) in the absorption bandedge. For an operation wavelength ($\lambda_0$) near the bandedge, the application of an electric field leads to an increase in the absorption coefficient.

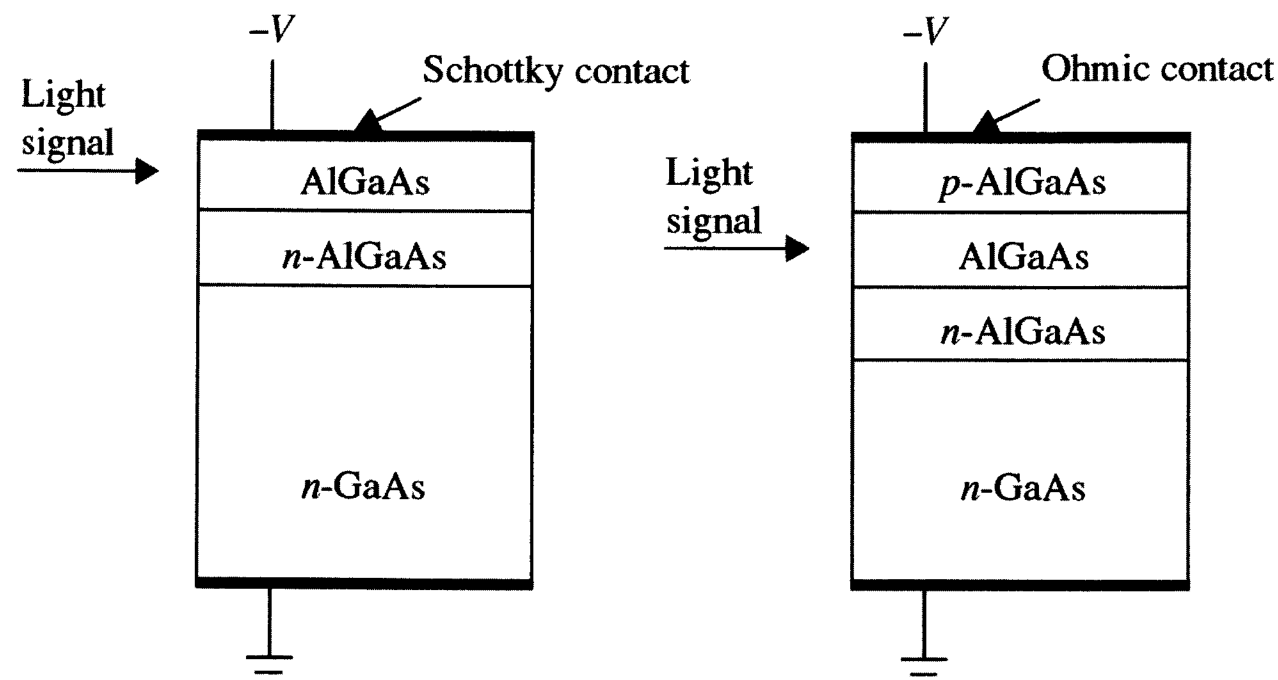



**Figure 9.16** Schematic drawings of EAMs based on Schottky diodes or PIN structures.