Dispersion in Weakly Guiding Fibers

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Optical signals transmitted through cladded glass fibers are subject to delay distortion because of (1) dispersion in the material, (2) dispersion caused by the waveguide characteristic, and (3) delay differences between modes. We isolate these effects and evaluate their significance for cases of practical interest. These concern fibers in which the refractive index of the cladding is only slightly lower than that of the core.

I. Introduction

A fiber waveguide consists of a thin central glass core surrounded by a glass cladding of slightly lower refractive index. In general, waveguides of this kind support many modes, which propagate at different velocities. Since this causes signal distortion over long distances, fibers which transmit only a limited number of modes are of special interest. Most modes can be suppressed by making the core thin and the index difference between core and cladding small. Reducing the index difference decreases the guidance for the remaining modes. In general, however, an index difference of a few parts in a thousand is sufficient to provide guidance around bends with radii of a few feet.

The dispersion in fiber waveguides has been the subject of various measurements[1–4] and computer evaluations.[5–8] These have provided data for certain fiber materials and operating conditions. Since both the material dispersion and the waveguide characteristics influence dispersion in a complicated way, no general conclusions have been drawn from these data.

We show here that for the case of the weakly guiding fiber (small index difference), simple relations obtain. Material and waveguide effect can be separated. General formulas are described which should be helpful for the design of fiber communication systems.

II. Group Delay

Direct detection of intensity-modulated light recognizes dispersion effects merely in the envelope of the light signal. This envelope propagates at the group velocity. For the sake of convenience we consider here the inverse of this velocity, which has the physical significance of a delay per unit length. We call this quantity the *group delay*. It is related to the propagation constant β of the carrier by

$$\tau = (1/c)(d\beta/dk), \tag{1}$$

where c is the vacuum speed of light and $k = 2\pi/\lambda$ the vacuum wavenumber. A plane wave carrier in a dispersive medium with refractive index n has

$$d\beta/dk = d(kn)/dk = N, \tag{2}$$

where N is called the *group index* of the material.

Consider now a fiber with core index n_1 and cladding index n_2. The propagation constant of any mode of this structure is limited within the interval $n_1k \geq \beta \geq n_2k$. Let the core radius be a. If we then define parameters[9]

$$u = a(k^2n_1^2 - \beta^2)^{\frac{1}{2}} \tag{3}$$

and

$$w = a(\beta^2 - k^2n_2^2)^{\frac{1}{2}}, \tag{4}$$

the mode field can be expressed by Bessel functions $J(ur/a)$ for radii $r < a$ and modified Hankel functions $K(wr/a)$ for $r > a$. Matching these fileds at $r = a$ yields functions $u(w)$ which characterize the various modes. It is apparent from Eq. (10) in Ref. 10 that the functions $u(w)$ are universal functions of the weakly guiding fiber independent of specific materials and dimensions. To derive the propagation constant, let us introduce two similarly universal parameters

$$v = (u^2 + w^2)^{\frac{1}{2}} \tag{5}$$

and

$$b = 1 - u^2/v^2. \tag{6}$$

From Eqs. (3) and (4) we obtain

$$v = ak(n_1^2 - n_2^2)^{\frac{1}{2}} \tag{7}$$

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and

$$b = (\beta^2/k^2 - n_2^2)/(n_1^2 - n_2^2). \tag{8}$$

Equation (8) then yields the propagation constant

$$\beta = k(n_2^2 + (n_1^2 - n_2^2)b)^{\frac{1}{2}}. \tag{9}$$

Plots and analytic solutions for $b(v)$ are given in Ref. 10.

The derivative of β is

$$d\beta/dk = \{n_2N_2 + [b + \tfrac{1}{2}v(db/dv)](n_1N_1 - n_2N_2)\} / [n_2^2 + (n_1^2 - n_2^2)b]^{\frac{1}{2}}, \tag{10}$$

where N_1 and N_2 are the group indices of core and cladding, respectively. Since the index difference is small, we can approximate Eq. (10) by

$$d\beta/dk = N_2 + (N_1 - N_2)b + {}^1/_2[(n_1/n_2)N_1 - N_2]v(db/dv). \tag{11}$$

Sufficiently far from cutoff, where $b \approx 1$ and $v\, db/dv \approx 0$, Eq. (11) reduces to $d\beta/dk \approx N_1$; this indicates that the group delay of a mode far from cutoff approaches that of a plane wave in the core material.

In most weakly guiding fibers, not only the indices but also the dispersion characteristics of core and cladding are very similar. In this case, we may use the approximation

$$(n_1 - n_2)/n_2 \approx (N_1 - N_2)/N_2 \ll 1, \tag{12}$$

which leads to

$$d\beta/dk = N_2 + (N_1 - N_2)[d(vb)/dv]. \tag{13}$$

The indices n and N for a few potential fiber materials are plotted in Fig. 1. Figure 2 shows $b(v)$ and $d(vb)/dv$ for the HE_{11} mode. Plots for modes of higher order may be found in Ref. 10.

If we have M modes and number these in the sequence of their cutoff values on the v scale, we can write approximately

$$d(vb)/dv \approx 1 + (u^2/v^2) \approx 1 + (m/M) \tag{14}$$

for the mth mode, excluding modes very close to cutoff ($m \approx M$)[10]. Thus, with Eqs. (2), (13), and (14), the delay of the mth mode is approximately

$$T = (1/c)[N_1 + (N_1 - N_2)(m/M)]. \tag{15}$$

Modes of low order (far away from cutoff) cause a delay of roughly N_1/c, the retardation of a plane wave traveling in the core material. The higher the mode number, the slower the energy transport, as expected. The difference in delay can be as large as $(N_1 - N_2)/c$ or 30 nsec/km for an index difference of 1%. The actual time dispersion on account of mode delay in practical fibers may well be smaller if certain modes are not excited, strongly attenuated, or convert to different modes because of imperfections. Equation (15) is valid for multimode fibers with large mode volumes. If only a few or just one mode propagates, the more general Eqs. (11) or (13) must be used to calculate τ.

III. Frequency Dependence of the Group Delay

Many light sources considered for optical communication cover a fairly wide optical frequency band. Light-emitting diodes, for example, have a relative band (bandwidth divided by the center frequency) of about 4%, GaAs lasers have up to 0.1%, and Nd:YAlG lasers roughly 0.01%. Signals modulating the intensity of such carriers can suffer distortion in long fibers because of differences in (group) delay between the various optical frequency components present. Consider, for example, a 1-nsec pulse from a light-emitting diode. After propagating through a long fiber, the optical components which constitute the pulse arrive at different times. If the optical band is not too wide, the time delay per unit frequency and unit length of fiber is approximately $d\tau/df$ with τ from Eq. (2). Let F be the optical center frequency and Δ the relative band. We then have a total delay

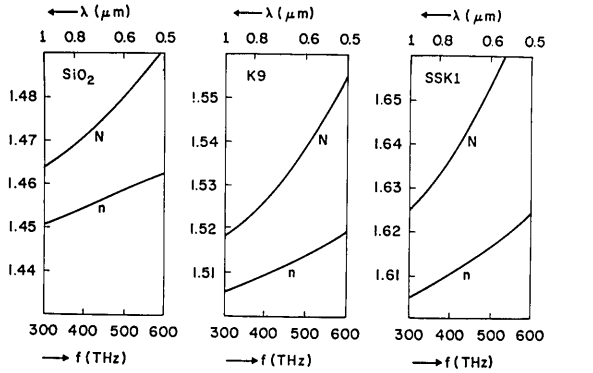

Fig. 1. Reractive index n and group index $N = d(kn)/dk$ as a function of frequency for fused silica (SiO_2) and the Schott glasses K9 and SSK1.

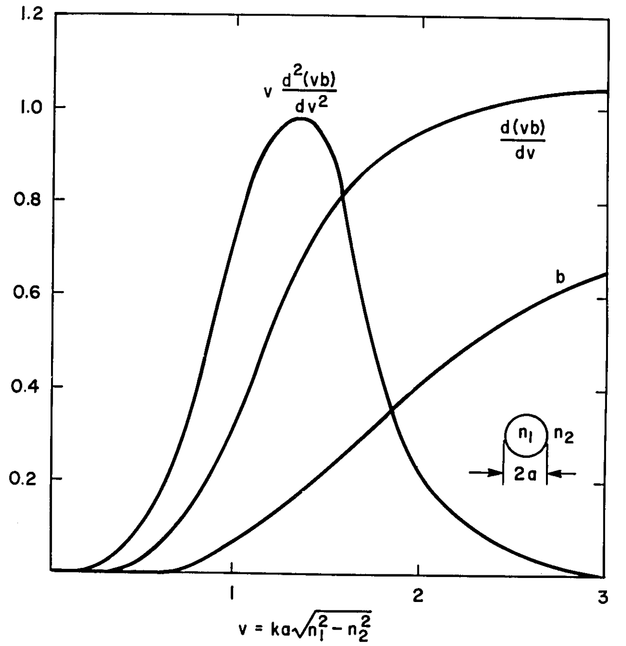

Fig. 2. Waveguide parameter b and its derivatives $d(vb)/dv$ and $vd^2(vb)/dv^2$ as a function of the normalized frequency v.

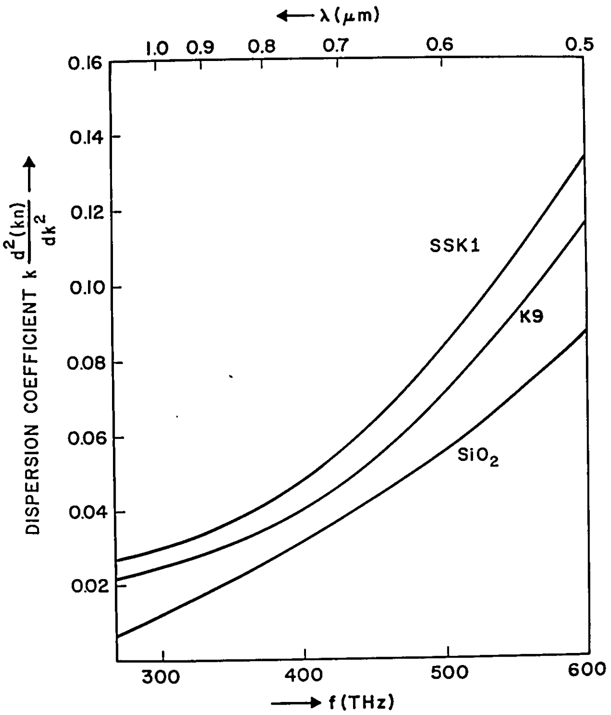

Fig. 3. Dispersion coefficient $kd^2(kn)/dk^2$ as a function of frequency for fused silica (SiO_2) and the Schott glasses K9 and SSK1.

$$s = \Delta F(d\tau/df) = \Delta(1/c)k(d^2\beta/dk^2) \tag{16}$$

per unit length, caused by the frequency dependence of the group delay. Differentiation of Eq. (13) with the help of Eq. (7) and the approximation (12) yields

$$k(d^2\beta/dk^2) = k(dN_2/dk) + k[d(N_1 - N_2)/dk][d(vb)/dv] + [(N_1^2 - N_2^2)/(n_1 + n_2)]v[d^2(vb)/dv^2]. \tag{17}$$

Sufficiently far from cutoff, where $d(vb)/dv \approx 1$ and $vd^2(vb)/dv^2 \approx 0$, Eq. (17) reduces to $kd^2\beta/dk^2 = kdN_1/dk$, which is the dispersion coefficient of the core material. Dispersion coefficients for a few potential fiber materials are plotted in Fig. 3.

In most glasses, $kdn/dk \ll 1$, which permits us to write a somewhat simplified approximation for Eq. (17). Introducing this into Eq. (16) yields

$$s = (\Delta/c)\{k(dN_2/dk) + (N_1 - N_2)v[d^2(vb)/dv^2]\}. \tag{18}$$

The second term in Eq. (18) characterizes the influence of the waveguide. The derivative $vd^2(vb)/dv^2$ is shown in Fig. 2 for the dominant mode (HE_{11}). It changes from 0.2 to 0.1 in the range of single-mode operation ($v = 2.0\ldots2.4$). Thus, if the index difference is 1% or less, the delay distortion in the single-mode fiber is determined almost solely by the material. With the data from Fig. 3, we find s to be of the order of 4 nsec/km for light-emitting diodes, 0.1 nsec/km for GaAs lasers, and 0.01 nsec/km for Nd:YAlG.

For modes of higher order, the derivatives $vd^2(vb)/dv^2$ can be obtained from the plots in Ref. 10. They are largest in magnitude at or near cutoff and decrease as v increases. When considering a fiber which propagates a large number of modes, we may neglect the few modes that are near cutoff. Then, for the majority of modes, we can use the approximations[10] that led to eq. (14) and find

$$v/[d^2(vb)/dv^2] \approx -2(u^2/v^2) \approx -2(m/M). \tag{19}$$

Thus, Eq. (18) becomes

$$s \approx \Delta(1/c)[k(dN_2/dk) - 2(m/M)(N_1 - N_2)]. \tag{20}$$

The negative waveguide characteristic compensates to a certain extent for the positive material effect.

The results Eqs. (18) and (20) are based on the assumption that the input pulse can be decomposed into several optical frequency components, each forming a pulse of the same shape as the composite pulse. As we assume the group delay to be approximately proportional to the frequency within the carrier band, the output pulse is a convolution of the pulse shape with the spectral profile of the carrier. In general, both shapes are independent and arbitrary. Yet if they are both gaussian with widths T and ΔF, respectively, the output pulse has a gaussian shape as well. Its width is

$$(T^2 + L^2s^2)^{\frac{1}{2}} \tag{21}$$

with s from Eq. (18) or (20). Here L is the length of the fiber.

The simple picture used above holds only for the case that the spectrum of the pulse envelope is negligible compared to the carrier spectrum. In many cases, this is not true. A perfectly mode-locked laser, for example, generates pulses that are so short that their spectrum covers the entire gain profile or more. This case of the bandwidth-limited pulse has been considered by Garrett and McCumber.[11] Assuming a gaussian pulse shape, they calculate a width

$$T^2 + \{[(L/T)(d^2\beta/d\omega^2)]^2\}^{\frac{1}{2}} \tag{22}$$

for the gaussian output pulse, where $\omega = 2\pi f$. The same formula can be obtained from Eqs. (21) and (16) if we set $\Delta F = (2\pi T)^{-1}$. This agreement is a good indication that Eq. (16) and hence also Eqs. (18) and (20) are of general validity if we understand Δ as the relative band of the modulated optical signal.

IV. Conclusions

The dispersion of a dielectric waveguide mode is a function both of the material and of the waveguide characteristic. If the refractive index difference between core and cladding is small, the two effects can be separated. In multimode guides, the waveguide introduces delay between the various modes. If all modes are excited and reach the end with little attenuation or conversion, the dispersion of an input pulse can be as high as 30 nsec/km for every percent of index difference. The single mode guide, in general, suffers mostly from material dispersion. The temporal spread of a (pulsed) input signal depends on the bandwidth of this signal. For practical fiber materials, it is of the order of 1 nsec/km for every percent of relative bandwidth.

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