

Pin photodiode (PT10XGC)

$$P_{min} = -20 \text{ dBm}$$

$$R = 0.8 \text{ A/W}$$

$$\text{BER} = 10^{-12}, B = 9 \text{ GHz}$$

SMF28 optical fiber

$$\alpha = 0.2 \text{ dB/km}$$

CQF 938/500 DFB laser

$$P_t = 50 \text{ mW} = 17 \text{ dBm}$$

$$\lambda = 1550 \text{ nm}$$

$$\Delta f = 1 \text{ MHz}$$

Maximum distance

$$PB = 17 + 20 = 37 \text{ dB}$$

$$\frac{37 \text{ dB}}{0.2 \text{ dB/km}} = \boxed{185 \text{ km}}$$

We will also need the thermal noise

$$49.5 = \frac{[(0.8)(10^{-5})]^2}{\sigma_{th}^2}$$

$$\sigma_{th}^2 = \frac{[(0.8)(10^{-5})]^2}{49.5}$$

$$\sigma_{th}^2 = 1.29 \times 10^{-12}$$

Use an EDFA (CPA series from JDS Uniphase)

Flatness 1.5 dB

PMD 0.5 ps

NF 6.0 for $G = 23 \text{ dB}$

8.4 for $G = 13 \text{ dB}$

$$5 < G < 23 \text{ dB}$$

$$P_{sat} = 17 \text{ dBm}$$

Why is the noise figure gain dependent?

Gain is changed by increasing pump power, which increases the population inversion.

High gain \rightarrow higher N_{sp}

Where can we place the EDFA?

At the transmitter: Power Amplifier

At the receiver: Pre-amplifier

In the middle of the link

As a power amplifier the EDFA will not do much because $P_t = P_{sat}$

What is the best case (No EDFA noise)?

EDFA adds no noise

$$PB = 17 + 20 + 23 = 60 \text{ dB}$$

$$L = \frac{60}{.2} = 300 \text{ Km}$$

USE EDFA as a preamplifier

$$G_{ASE}^2 = 4R^2 G^2 P_{in} n_{sp} \chi hf \Delta f$$

We know everything except for $n_{sp} \chi$

The spec sheet provides noise figure NF

$$NF = \left(\frac{SNR_{in}}{SNR_{out}} \right) = \frac{SNR_{shot}}{SNR_{ase}} = \left(\frac{(R P_{in})^2}{2q R P_{in} \Delta f} \right) \frac{4R^2 G^2 P_{in} n_{sp} \chi hf \Delta f}{(R G P_{in})^2}$$

$$= \left(\frac{R P_{in}}{2q \Delta f} \right) \frac{4 n_{sp} \chi hf \Delta f}{P_{in}}$$

$$= R \left(\frac{hf}{q} \right) 2 n_{sp} \chi$$

$$R = \eta \frac{q}{hf}$$

$$NF = \eta \left(\frac{q}{hf} \right) \left(\frac{hf}{q} \right) 2 n_{sp} \chi$$

$$NF = (\eta) (2 n_{sp} \chi)$$

We just need η

$$R = 0.8 = \eta \frac{(1.55)}{1.24}$$

$$\eta = 0.64$$

$$10 \log_{10}(6) = (0.64) 2 n_{sp} \chi$$

$$n_{sp} \chi = \frac{10 \log_{10}(6)}{(2)(0.64)}$$

$$\boxed{n_{sp} \chi = 6} \quad \text{for high gain}$$

$$\text{for low gain } n_{sp} \chi = 7.22$$

Assume σ_{ASE}^2 is the dominant noise term

$$SNR = \frac{(RG P_{in})^2 \lambda}{4 R^2 G^2 P_{in} n_{sp} \lambda h c \Delta f}$$

$$SNR = \frac{P_{in} \lambda}{4 n_{sp} \lambda h c \Delta f}$$

$$P_{in} = (SNR) (4) n_{sp} \lambda \left(\frac{hc}{\lambda}\right) \Delta f$$

$$P_{in} = (49.5) (4) (6) \frac{(6.6 \times 10^{-34}) (3 \times 10^8)}{1.55 \times 10^{-6}} (9 \times 10^9)$$

$$P_{in} = 1.37 \mu W = -28.65 \text{ dBm}$$

$$L = \frac{17 + 28.65}{0.2}$$

$$L = 228 \text{ km}$$

a gain of 43 km
substantially shorter than the ideal of
300 km

Was the assumption valid?

$$\sigma_{ASE}^2 = 4R^2 G^2 P_{in} n_{sp} \lambda \frac{hc}{\lambda} \Delta f = 9.6 \times 10^{-10}$$

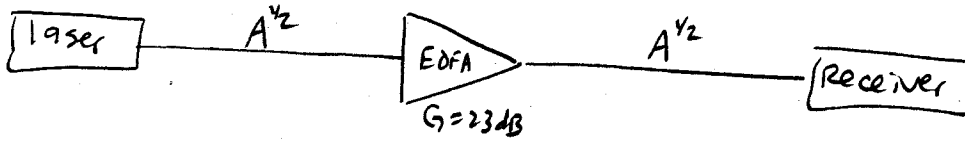
$$\sigma_{sh}^2 = 2q R G P_{in} = 6.98 \times 10^{-23}$$

$$\sigma_{th}^2 = 1.29 \times 10^{-12}$$

$$\text{SO } \sigma_{ASE}^2 \gg \sigma_{sh}^2$$

$$\sigma_{ASE}^2 \gg \sigma_{th}^2$$

Now place the EDFA in the middle of the link



The EDFA noise is independent of position

$$\sigma_{ASE}^2 = 4R^2G^2 P_{in} n_{sp} \times \frac{hc}{\lambda} \Delta f$$

$$= 4R^2G^2 (P_t A) n_{sp} \times \frac{hc}{\lambda} \Delta f$$

But the noise is attenuated by $A^{1/2}$

$$\sigma_{ASE}^2 = 4R^2G^2 (P_t A) n_{sp} \times \frac{hc}{\lambda} \Delta f A^{1/2}$$

Assume σ_{ASE}^2 dominates

$$SNR = \frac{(R P_t A G)^2}{4R^2G^2 (P_t A) n_{sp} \times \frac{hc}{\lambda} \Delta f A^{1/2}}$$

$$SNR = \frac{P_t A^{1/2}}{4 n_{sp} \times \frac{hc}{\lambda} \Delta f}$$

$$A^{1/2} = (SNR) 4 n_{sp} \times \frac{hc}{\lambda} \Delta f \left(\frac{1}{P_t}\right)$$

$$A = \left[(49.5) (4) (6) \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{1.55 \times 10^{-6}} \frac{(9 \times 10^9)}{50 \times 10^{-3}} \right]^2$$

$$A = 7.46 \times 10^{10} = 91.3 \text{ dB}$$

$$L = \frac{91.3}{0.2} = 456.5 \text{ km} > 300 \text{ km} \text{ so something is wrong}$$

check assumptions

$$\sigma_{sh}^2 = 29 R G P_{in} = 1.9 \times 10^{-27}$$

$$\sigma_{ASE}^2 = 4R^2G^2 P_{in} n_{sp} \times \frac{hc}{\lambda} \Delta f A^{1/2} = 7.16 \times 10^{-19}$$

$$\sigma_{th}^2 = 1.29 \times 10^{-12}$$

so $\sigma_{th}^2 > \sigma_{ASE}^2$

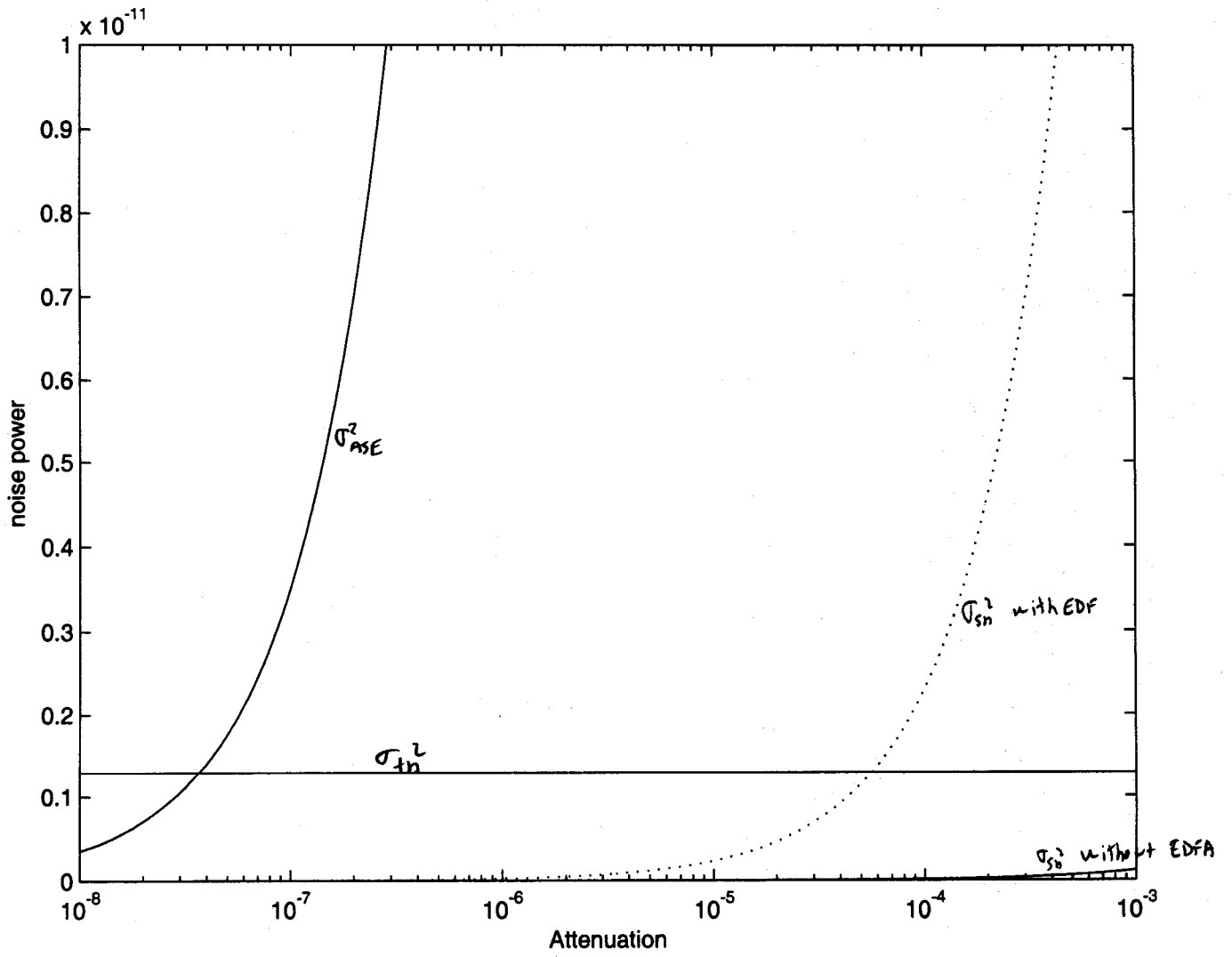
$$SNR = \frac{(R P_t A G)^2}{\sigma_{th}^2 + 4R^2 G^2 (P_t A) n_{sp} \times \frac{hc}{\lambda} \Delta f A^{1/2}}$$

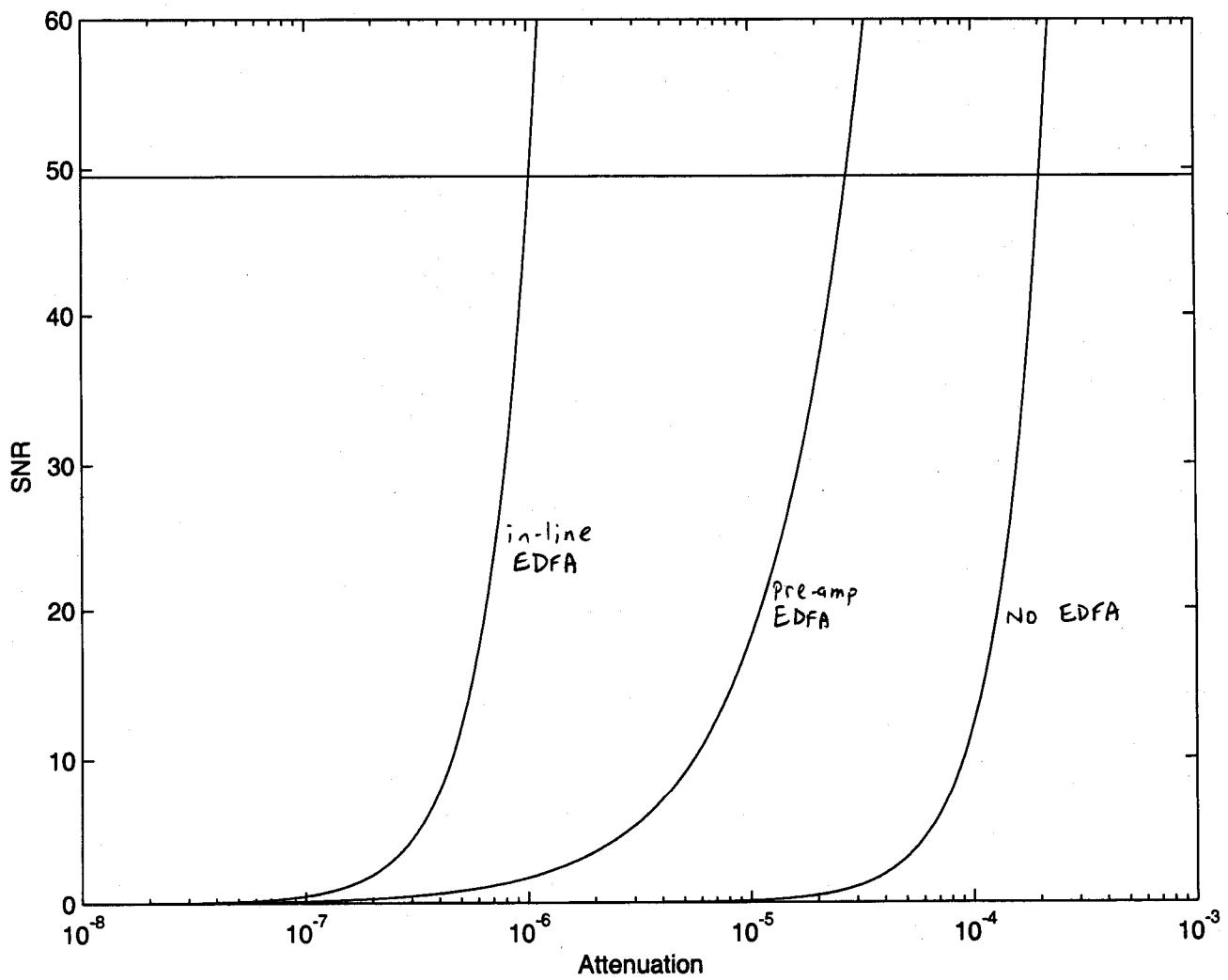
$$(R^2 P_t^2 G^2) A^2 - SNR (4) R^2 G^2 P_t n_{sp} \times \frac{hc}{\lambda} \Delta f A^{1/2} - SNR \sigma_{th}^2 = 0$$

Solve for A

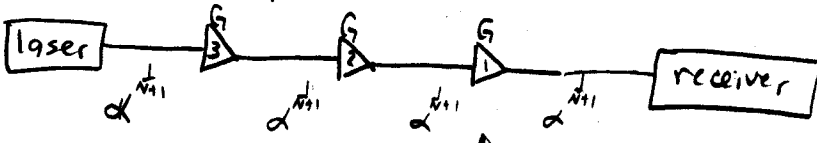
$$A = 1.023 \times 10^{-6} = 59.9 \text{ dB}$$

$$L = \frac{59.9}{0.2} = 299.5 \text{ km} \quad \text{very close to ideal length}$$





If multiple amplifiers are used



$$\sigma_{ASE}^2 = 4 (R P_t \alpha G^N) \left(R n_{sp} \times \frac{hc}{\lambda} \Delta f G \right)$$

$$\sigma_{ASE1}^2 = \sigma_{ASE}^2 \alpha^{\frac{1}{N_{th1}}}$$

$$\sigma_{ASE2}^2 = \sigma_{ASE}^2 \left(\alpha^{\frac{2}{N_{th1}}} \right) G$$

$$\sigma_{ASE3}^2 = \sigma_{ASE}^2 \alpha^{\frac{3}{N_{th1}}} G^2$$

...

$$\begin{aligned} \sigma_{ASE, TOT}^2 &= \frac{\sigma_{ASE}^2}{G} \left[G \alpha^{\frac{1}{N_{th1}}} + (G \alpha^{\frac{2}{N_{th1}}})^2 + (G \alpha^{\frac{3}{N_{th1}}})^2 + \dots (G \alpha^{\frac{N}{N_{th1}}})^{N-1} \right] \\ &= \sigma_{ASE}^2 \alpha^{\frac{1}{N_{th1}}} \left[1 + G \alpha^{\frac{1}{N_{th1}}} + (G \alpha^{\frac{1}{N_{th1}}})^2 + \dots (G \alpha^{\frac{1}{N_{th1}}})^{N-1} \right] \end{aligned}$$

$$\text{let } G \alpha^{\frac{1}{N_{th1}}} = r$$

$$\sigma_{ASE, TOT}^2 = \sigma_{ASE}^2 r (1 + r + r^2 + \dots + r^{N-1})$$

$$1 + r + r^2 + \dots = \frac{1}{1-r} \quad \text{if } |r| < 1$$

$$r^N + r^{N+1} + \dots = r^N (1 + r + r^2 + \dots) = r^N \frac{1}{1-r}$$

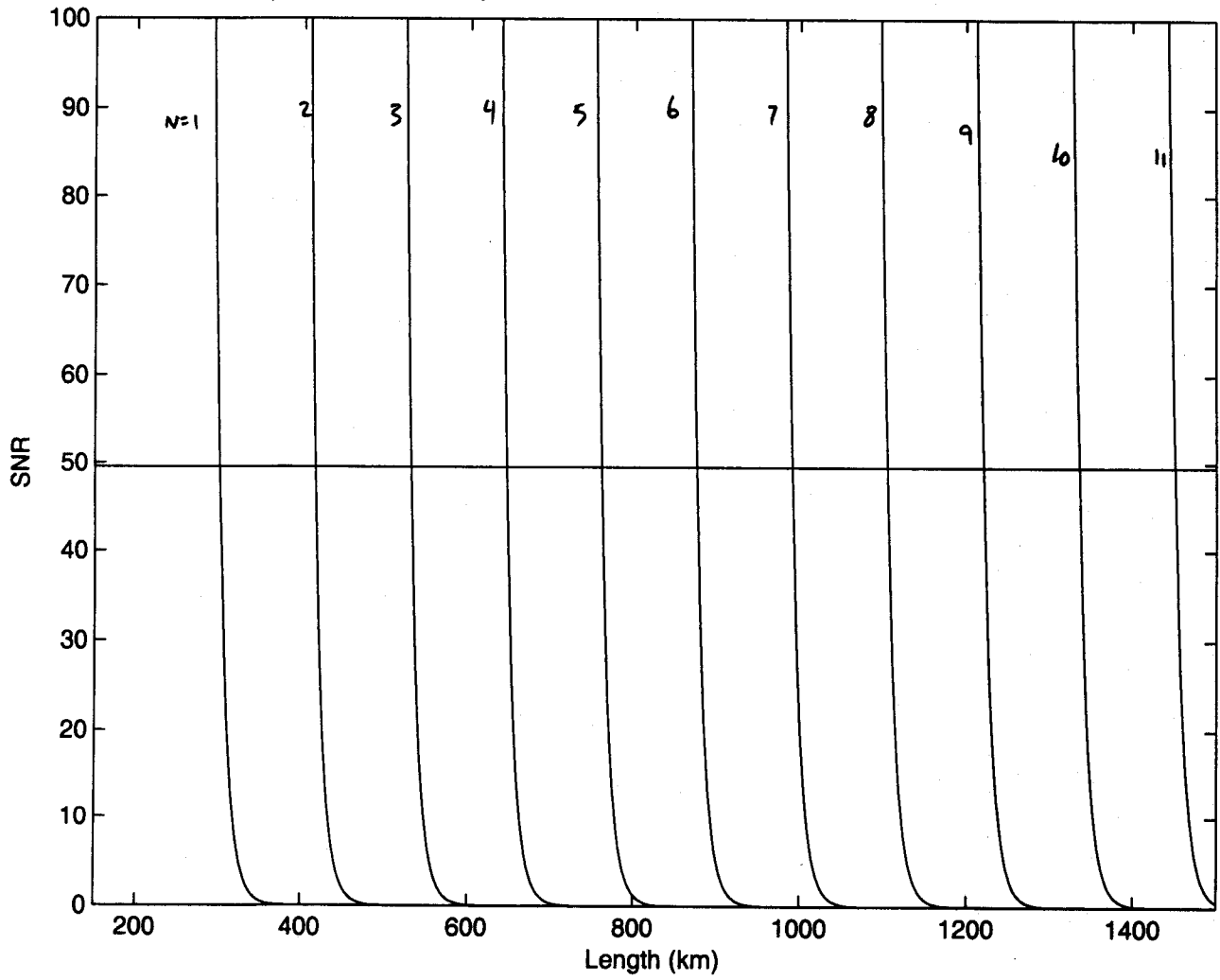
$$1 + r + r^2 + \dots + r^N = \frac{1}{1-r} - \frac{r^N}{1-r} = \frac{1-r^N}{1-r}$$

$$\sigma_{ASE, TOT}^2 = 4 (R P_t \alpha G^N) \left(R n_{sp} \times \frac{hc}{\lambda} \Delta f \right) (G \alpha^{\frac{1}{N_{th}}}) \left(\frac{1 - G^N \alpha^{\frac{N}{N_{th}}}}{1 - G \alpha^{\frac{1}{N_{th}}}} \right)$$

$$SNR_{(m)} = \frac{(R \alpha P_t G^N)^2}{2q R \alpha P_t G^N \Delta f + \sigma_{th}^2 + \sigma_{ASE, TOT}^2}$$

$$\underbrace{2q R \alpha P_t G^N \Delta f}_{\text{shot}} + \underbrace{\sigma_{th}^2}_{\text{thermal}} + \underbrace{\sigma_{ASE, TOT}^2}_{\text{ASE}}$$

with in-line EDFAs



$$\alpha = 0.2 \text{ dB/km}$$