Example

Calculate Dispersion limited length using SMF28 optical fiber with the following lasers

(a) LED $\lambda = 1310nm$ $\Delta\lambda = 30nm$
(b) laser $\lambda = 1550nm$ $\Delta\lambda = 0.001nm$

use $B = 10Gbps$
For the fiber use $\lambda_0 = 1310nm$
$S_0 = 0.092$

(a) $\Delta\lambda_{laser} \gg \Delta\lambda_{modulation}$

$$\Delta T = (D_{intra})\,\Delta\lambda\, L + \frac{1}{2}\left(\frac{dD_{intra}}{d\lambda}\right)\Delta\lambda^2 L$$

at $\lambda_0 = 1310nm$ $D_{intra} = 0$

$$\frac{dD}{d\lambda} = \frac{S_0}{4}\left[1 + 3\frac{\lambda_0^4}{\lambda^4}\right] \quad \text{at } \lambda = \lambda_0$$

$$= S_0$$

$$\Delta T = \left(\frac{1}{2}\right)(S_0)(30)^2 L = \frac{1}{4B}$$

$$L = \frac{2}{(S_0)(30)^2(4B)}$$

$$= \frac{2}{\left(0.092 \times 10^{-12}\ \frac{s}{nm^2\,km}\right)(30)^2\,nm^2\,(4)(10^9\,s^{-1})}$$

$$\boxed{L = 0.6\,km}$$

(b) $\Delta T = (D_{intra}\,\Delta\lambda\, L) = \frac{1}{4B}$

$$L = \frac{1}{4\,D_{intra}\,\Delta\lambda\,B}$$

$$D_{intra} = 0.092\left[1550 - \frac{1310^4}{1550^3}\right] = 17.46\ \frac{ps}{km \cdot nm}$$

For modulation $\Delta f = 2B$

$$f = \frac{c}{\lambda}$$

$$\frac{df}{d\lambda} = -\frac{c}{\lambda^2} \qquad \Delta\lambda = \frac{\lambda^2}{c}\Delta f$$

$$\Delta\lambda = \frac{(1550\times10^{-9})^2}{3\times10^8}\,(2)(10^{10})$$

$$\Delta\lambda_{mod} = 0.16\ nm \gg \Delta\lambda_{laser}$$

$$L = \frac{1}{(4)(17.46\times10^{-12})(0.16)(10^{10})}$$

$$\boxed{L = 8.95\ Km}$$