1. Why is it impractical to create an EDFA using the doping denoted as Silicate L22 to amplify an optical signal at 980nm? Be sure to use an equation to prove your answer. (It has nothing to do with the emission or absorption cross-sections.)

The gain equation is

\[ g = \frac{\text{Pe} \cdot N_t \cdot W_p - \gamma_p}{W_p + 2\gamma_s + \gamma_Tsp} \]

for gain we need \( W_p > \frac{1}{T_{sp}} \)

\[ \frac{\text{Pe} \cdot P_p}{h \cdot f \cdot A} > \frac{1}{0.001 \cdot \mu \text{W}^{-2}} \]

\[ P_p > \frac{h \cdot c \cdot A}{\eta \cdot \lambda \cdot (0.001 \cdot \mu \text{W}^{-2})} \]

The pump wavelength needs to be lower than 980nm. Let's use \( \lambda_p = 800\text{nm} \)

\[ P_p > \frac{(6.6 \times 10^{-34}) \cdot (3 \times 10^8) \cdot 50 \times 10^{-12}}{4 \times 10^{-16} \cdot (0.8 \times 10^3) \cdot 0.001 \times 10^{-2}} \]

\[ P_p > 309\text{W} \] which is too high to be practical
2. What is the minimum pump power required for gain for the following EDFAs?
   a. Silicate L22 pumped at a wavelength of 980nm.
   b. Fluorozirconate F88 pumped at a wavelength 1480nm.

\[ \frac{\sigma, \ p_p}{h \cdot f \cdot A} > \frac{1}{\tau_p} \]
\[ P_p > \frac{h \cdot c \cdot A}{\sigma \cdot \lambda \cdot \tau_p} \]

(a) Silicate L22 at \( \lambda_p = 980 \text{nm} \)
\[ P_p > \frac{(6.6 \times 10^{-14} \times 3 \times 10^3 \times 50 \times 10^{-12})}{(10 \times 10^{-26})(14.5 \times 10^{-3})(0.98 \times 10^{-6})} \]
\[ P_p > 6.97 \text{ mW} \]

(b) \[ P_p > \frac{(6.6 \times 10^{-14} \times 3 \times 10^3 \times 50 \times 10^{-12})}{(2 \times 10^{-25})(1.48 \times 10^{-6})(9.4 \times 10^{-3})} \]
\[ P_p > 3.6 \text{ mW} \]
**Figure 17.13** Absorption cross section of \( \text{Er}^{3+} \) at 800 nm.

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**Figure 17.14** Absorption cross section of \( \text{Er}^{3+} \) at 980 nm.

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Figure 17.18 Absorption cross section of $\text{Er}^{3+}$ at 1450 nm.

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Figure 17.16 Emission cross section of $\text{Er}^{3+}$ at 1540 nm.

Source: Reprinted, by permission, from Miniatura. "Erbium-Doped Glasses for Fiber Amplifiers at 1550 nm."
3. A Fluorozirconate F88 EDFA is pumped at a wavelength of $\lambda_p=1480\text{nm}$, with a pump power equal to 5 times the minimum pump power required for gain. If the amplifier has a length of $L=5\text{m}$, what is the total gain? Assume the signal wavelength is $\lambda_s=1554\text{nm}$.

$$\lambda_p = 1480\text{nm}$$
$$W_P = 5/\gamma_{sp}$$
$$L = 5\text{m}$$
$$\gamma_s = 1554\text{nm}$$
$$N_s = 8 \times 10^{18} \text{ cm}^{-3}$$
$$\Delta = 4 \times 10^{-21}$$

$$g^* = (4 \times 10^{-11}) \text{ cm}^2 (8 \times 10^{18}) \text{ cm}^{-3}$$
$$g^* = 0.032 \text{ cm}^{-1} = 3.2 \text{ m}^{-1}$$

$$g = g^* \left( \frac{W_P - \gamma_{sp}}{W_P + \gamma_{sp}} \right)$$

$$g = 3.2 \left( \frac{5/\gamma_{sp} - \gamma_{sp}}{5/\gamma_{sp} + \gamma_{sp}} \right)$$

$$g = 3.2 \left( \frac{4}{6} \right)$$
$$g = 2.13 \text{ m}^{-1}$$

$$= (4.34)(2.13)$$
$$= 9.26 \text{ dB/m}$$

$$G = (5)(9.26)$$

$$G = 46.3 \text{ dB}$$
Figure 17.16 Emission cross section of Er\textsuperscript{3+} at 1540 nm.

Source: Reprinted, by permission, from Minisculeco. "Erbium-Doped Glasses for Fiber..."
4. An amplifier is constructed with a peak gain of $G = 30 \text{dB}$. Let's define the amplifier wavelength range as the wavelength range over which the normalized gain variation is less than 10dB.

a. Which of the amplifier materials (Silicate L22 or Fluorozirconate F88) has the largest wavelength range?

b. What is the wavelength range for the best material?

\[
G = 30 \text{dB} \\
\Delta G = 10 \text{dB}
\]

\[
G = \sigma_e N \left( \frac{W_p - V_{sp}}{W_p + V_{sp}} \right)
\]

\[
G_{\text{dB}} = 4.34 \sigma_e N \left( \frac{W_p - V_{sp}}{W_p + V_{sp}} \right)
\]

\[
G = 4.34 \sigma_e N L \left( \frac{W_p - V_{sp}}{W_p + V_{sp}} \right) = \left[ 4.34 N L \left( \frac{W_p - V_{sp}}{W_p + V_{sp}} \right) \right] \sigma_e
\]

\[
G = A_0 \sigma_e \\
30 = A_0 \sigma_{e, \text{max}} \\
20 = A_0 \sigma_{e, \text{min}} \\
\sigma_{e, \text{min}} = \frac{20}{30}
\]

\[
\sigma_{e, \text{max}} = 8.1 \times 10^{-21} \quad \text{L22} \\
\sigma_{e, \text{min}} = 5.4 \times 10^{-21} \quad \text{F88}
\]

\[
\lambda_{\text{min}} = 1534 \text{nm} \quad \text{L22} \\
\lambda_{\text{max}} = 1550 \text{nm} \\
\Delta \lambda = 16 \text{nm}
\]

(a) Fluorozirconate F88

(b) $\Delta \lambda = 40 \text{nm}$
Figure 17.15  Absorption cross section of Er$^{3+}$ at 1450 nm.

Source: Reprinted, by permission, from Miniscalco, "Erbium-Doped Glasses for Fiber Amplifiers at 1550 nm," Figure 7 [10]. © 1991 by IEEE.

Figure 17.16  Emission cross section of Er$^{3+}$ at 1540 nm.

Source: Reprinted, by permission, from Miniscalco. "Erbium-Doped Glasses for Fiber
Figure 17.13  Absorption cross section of Er$^{3+}$ at 800 nm.

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Figure 17.14  Absorption cross section of Er$^{3+}$ at 980 nm.

Source: Reprinted, by permission, from Miniscalco, "Erbium-Doped Glasses for Fiber Amplifiers at 1500 nm," Figure 12 [10], © 1991 by IEEE.
(a) \[ g = \frac{\alpha_e N_T \frac{W_p - Y_{sp}}{W_p + Y_{sp}}}{W_p + Y_{sp}} \]

\[ G = \alpha_e N_T \left( \frac{m-1}{m+1} \right) \left( 4.34 L \right) \]

Assume \( W_p = m/\beta_p \)

\[ G = \alpha_e N_T \left( \frac{m-1}{m+1} \right) \left( 4.34 L \right) \]

\[ W_p = \frac{m}{\beta_p} \]

\[ \beta_p = \frac{\alpha_e \lambda_p}{h c A} \]

Pumping Efficiency \( \frac{G}{P_p} = \frac{\alpha_e N_T \left( \frac{m-1}{m+1} \right) \left( 4.34 L \right) \alpha_e \beta_p \lambda_p}{m h c A} \)

Maximize this \[ \lambda_p \]

Find best Pump Wavelength

For Silicate \( \lambda_p \)

- 1.191nm
- 1.5 \times 10^{-3} \text{ m} \text{ } \text{m}^2
- 2.2 \times 10^{-5} \text{ m} \text{ } \text{m}^3

- 0.980nm
- 1.5 \times 10^{-3} \text{ m} \text{ } \text{m}^2
- 9.8 \times 10^{-5} \text{ m} \text{ } \text{m}^3

For Fluoro Zirconate F88 \( \lambda_p \)

- 1.500nm
- 4.25 \times 10^{-21} \text{ m} \text{ } \text{m}^3

- 9.75nm
- 2.2 \times 10^{-22} \text{ m} \text{ } \text{m}^3

Material \( \alpha_e \) \( \beta_p \) \( \lambda_p \) \( \Sigma \) \( \tilde{\alpha} \)

Silicate \( 8.1 \times 10^{-21} \) \( 14.5 \) \( 2.2 \times 10^{-31} \) \( 2.6 \times 10^{-57} \text{ m}^3 s \)

Fluoro. \( 5.5 \times 10^{-21} \) \( 9.4 \) \( 6.4 \times 10^{-31} \) \( 3.3 \times 10^{-57} \text{ m}^3 s \)

Best design: Fluoro Zirconate F88, \( \lambda_p = 1.500 \text{ nm}, \lambda_s = 1.534 \text{ nm} \)

(b) Pump efficiency \( \frac{G}{P_p} = \left( \frac{3.3 \times 10^{-57} \text{ m}^3}{5} \right) \left( \frac{4}{4} \right) \left( 4.34 \text{ (5)} \right) \left( 8 \times 10^{-29} \text{ m}^3 \text{ } \text{s} \right) \left( 6.6 \times 10^{-17} \text{ m}^3 \right) \left( 1500 \text{ nm} \right)^2 \)

\[ \text{dB} \times 1 \text{W} \text{/} 10^3 \text{nm} = 7.9 \text{ dB/mW} \]
What is the gain of a silica L22 amplifier if $v_p = 980 \text{ V/m}$, $h_s = 1550 \text{ mm}$, $P_p = 10 \text{ mW}$, $L = 5 \text{ m}$, $P_i = 10 \text{ mW}$?

$\sigma_e = 3 \times 10^{-21} \text{ cm}^2$

$\sigma_a = 9 \times 10^{-22} \text{ cm}^2$

$W_p = \frac{\sigma_a P_p A_p}{h c A} = \frac{(9 \times 10^{-22})(10 \times 10^{-2}) (980 \times 10^{-2})}{(6.6 \times 10^{-19})(3 \times 10^{-8})(50 \times 10^{-15})} = 89.1$

$W_s = \frac{\sigma_e P_{in} A_s}{h c A} = \frac{(3 \times 10^{-24})(100 \times 10^{-4})(1550 \times 10^{-2})}{(6.6 \times 10^{-19})(3 \times 10^{-8})(50 \times 10^{-15})} = 4.70$

$g = \sigma_e N_f \left( \frac{W_p - V_{Tp}}{W_p + 2W_s + V_{Tp}} \right) = (3 \times 10^{-25})(8 \times 10^{-24}) \left( \frac{89.1 - 69.0}{89.1 + 9.4 + 69.0} \right)$

$g = 0.0238 \text{ m}^{-1}$

$J_{48} = 1.25 \text{ dB/m}$

$G_{dB} = 1.25 \times 5$

$G_{dB} = 6.26 \text{ dB}$
7. An optical receiver has the following specifications: \( R = 0.9 \text{ A/W} \), \( P_{\text{min}} = -20 \text{dBm} \) with \( B = 10 \text{Gb/s} \) and \( \text{BER} = 10^{-12} \). The optical fiber has an attenuation of \( \alpha = 0.2 \text{ dB/km} \) (this attenuation includes all splice losses), the transmitted optical power minus all other losses (coupling, loss margin, etc.) is \( 20 \text{mW} \), and the wavelength is \( \lambda = 1550 \text{nm} \). Ignore any dispersion limits in this problem.
   a. What is the maximum link length?
   b. What is the maximum link length if an EDFA is used as a pre-amplifier (right before the receiver)? The EDFA has the following specifications \( G = 30 \text{dB} \), \( \text{NF} = 7 \text{dB} \).
   c. What is the maximum link length if an EDFA is used near the middle of the optical communications link? The EDFA has the following specifications \( G = 30 \text{dB} \) and \( \text{NF} = 7 \text{dB} \).
   d. What is the maximum link length if 4 EDFAs are used? The EDFA has the following specifications \( G = 30 \text{dB} \) and \( \text{NF} = 7 \text{dB} \).

\[
L = \frac{13 + 20}{0.2} \approx 165 \text{km}
\]

(b) The best case is \( L = \frac{P_{\text{t}} - P_{\text{min}} + G}{\alpha} = \frac{13 + 20 + 30}{0.2} = 315 \text{km} \)

\[
\sigma_{\text{ASE}}^2 = 4R^2G^2\eta P_{\text{in}}n_{sp}\times hf \Delta f
\]

we need \( n_{sp} \)

\[
\text{NF} = \left( \frac{SNR_{\text{in}}}{SNR_{\text{out}}} \right) = \left( \frac{SNR_{\text{shot}}}{SNR_{\text{ASE}}} \right) = \left( \frac{(R \text{Pin})^2}{(R \text{Pin}\gamma / 2)} \right) \times \frac{4R^2G^2\eta P_{\text{in}}n_{sp}\times hf \Delta f}{(R \text{Pin}\gamma / 2)^2}
\]

\[
NF = \frac{4Rn_{sp}hf}{2q} = \frac{2n_{sp}}{R} (\frac{hf}{R})^2
\]

\[
NF = 2n_{sp} (\frac{hf}{R}) \eta (\frac{R}{hf})
\]

\[
\eta (\frac{R}{hf}) = \eta \frac{1.25}{1.24} = 0.9
\]

\[
\eta = 0.72
\]

\[
NF = 10^{0.7} = (n_{sp} \times 0.72)
\]

\[
n_{sp} (0.72) = 6.96
\]
Since the power at the receiver will be high \(\sigma_{\text{ASE}}^2 \gg \sigma_{\text{sh}}^2\).
Also assume \(\sigma_{\text{sh}}^2 < \sigma_{\text{ASE}}^2\).

\[
\text{SNR} = 49.5 = \frac{(R \cdot P_{\text{in}} \cdot G)^2}{4 \pi^2 \frac{G^2}{\delta f} \frac{P_{\text{in}} \cdot N}{\text{hf}}} \delta f
\]

\[
49.5 = \frac{P_{\text{in}}}{4 \cdot N_{\text{sp}} \cdot \text{hf} \cdot \delta f} \times \frac{(49.5)(4)(N_{\text{sp}})}{h \cdot c \cdot \delta f}
\]

\[
P_{\text{in}} = (49.5)(4)(6.96) \frac{(6.6 \times 10^{-34})(3 \times 10^8)}{1.55 \times 10^{-7}} \cdot 10^{10}
\]

\[
P_{\text{in}} = 1.76 \times 10^{-6} = -27.5 \text{ dBm}
\]

\[
L = \frac{13 + 27.5}{0.2}
\]

\[
L = 207.5 \text{ Km}
\]

check assumptions

\[
\sigma_{\text{ASE}}^2 = 4 \cdot R^2 \cdot G^2 \cdot \frac{P_{\text{in}} \cdot N_{\text{sp}} \cdot \frac{h \cdot c}{\lambda}}{\delta f}
\]

\[
= (4)(0.9)^2 \frac{(10^{15})^2}{(1.76 \times 10^{-6})(6.96)(6.6 \times 10^{-34})(3 \times 10^8)} \cdot 10^{10}
\]

\[
= 5.07 \times 10^{-6}
\]

\[
\sigma_{\text{sh}}^2 = 2q \cdot \frac{R^2 \cdot P_{\text{in}}}{c}
\]

\[
\leq (2)(1.6 \times 10^{19})(0.9)(10^3)(1.76 \times 10^{-6})
\]

\[
= 5.07 \times 10^{-22}
\]

For \(\sigma_{\text{sh}}^2\),

\[
49.5 = \frac{(R \cdot P_{\text{in}})^2}{\sigma_{\text{sh}}^2}
\]

\[
\sigma_{\text{sh}}^2 = \frac{(0.9)(10^{-22})^2}{49.5}
\]

\[
\sigma_{\text{sh}}^2 = 1.64 \times 10^{-22}
\]

\[
\sigma_{\text{ASE}}^2 \gg \sigma_{\text{sh}}^2
\]

\[
\sigma_{\text{ASE}}^2 \gg \sigma_{\text{sh}}^2
\]
(c) With EDFA in the middle of the link

\[
\text{SNR} = \frac{(RAGP_t)^2}{(\sigma_{\text{ASE} A^4}) + \sigma_{3b}^2 + \sigma_{j_b}^2}
\]

Since \( \sigma_{\text{ASE}}^2 \) is attenuated by \( A^4 \) it is probably closer to \( \sigma_{j_b}^2 \)

\[
49.5 = \frac{(12AGP_t)^2}{(4\pi^2 G^2 A P_t n_p \times \frac{h c}{f A^2}) + \sigma_{j_b}^2}
\]

This is a nonlinear equation in terms of the attenuation \( A \). It should be around \( (3\sigma_0)(0.1) = 60\text{dB} \) so around \( A = 10^{-6} \)

Solution \( A = 5.3317 \times 10^{-7} \)

\[
L = \frac{A}{0.2} = -62.73 \text{ dB}
\]

\[
L = 3.37 \text{ km}
\]
\[ I_{ph} = (R \cdot G^4 \cdot A \cdot P_t)^2 \]
\[ \sigma_{th}^2 = (R \cdot N \cdot E \cdot P)^2 \Delta f \]

Noise from each amplifier:
\[ \sigma_{ASE}^2 = 4 \left( R \cdot P \right) \left( R \cdot G \cdot n_{sp} \cdot x \cdot h \cdot f \right) \Delta f \]
\[ = 4 \cdot R^2 \cdot G^4 \cdot A \cdot P_t \cdot G \cdot n_{sp} \cdot x \cdot h \cdot f \cdot \Delta f \]
\[ = 4 \cdot R^2 \cdot G^5 \cdot A \cdot P_t \cdot n_{sp} \cdot x \cdot h \cdot f \cdot \Delta f \]

The noise from each amplifier is attenuated and amplified
\[ \sigma_{ASE1}^2 = \left( 4 \cdot R^2 \cdot G^5 \cdot A \cdot P_t \cdot n_{sp} \cdot x \cdot h \cdot f \cdot \Delta f \right) A^8 \]
\[ \sigma_{ASE2}^2 = \left( 4 \cdot R^2 \cdot G^5 \cdot A \cdot P_t \cdot n_{sp} \cdot x \cdot h \cdot f \cdot \Delta f \right) G \cdot A^{45} \]
\[ \sigma_{ASE3}^2 = \left( 4 \cdot R^2 \cdot G^5 \cdot A \cdot P_t \cdot n_{sp} \cdot x \cdot h \cdot f \cdot \Delta f \right) G^2 \cdot A^{34} \]
\[ \sigma_{ASE4}^2 = \left( 4 \cdot R^2 \cdot G^5 \cdot A \cdot P_t \cdot n_{sp} \cdot x \cdot h \cdot f \cdot \Delta f \right) G^3 \cdot A^{44} \]

\[ SNR = \frac{I_{ph}^2}{\sigma_{th}^2 + \sigma_{ASE1}^2 + \sigma_{ASE2}^2 + \sigma_{ASE3}^2 + \sigma_{ASE4}^2} \]

Plot and solve numerically to find
\[ SNR = 49 \]

\[ A = 5.27 \times 10^{-14} \]
\[ = -132.8 \text{ dB} \]

\[ L = 663.9 \text{ km} \]