1. Why is it impractical to create an EDFA using the doping denoted as Silicate L22 to amplify an optical signal at 980nm? Be sure to use an equation to prove your answer. (It has nothing to do with the emission or absorption cross-sections.)

The gain equation is

$$g = \sigma_e N_t \frac{W_p - 1/\tau_{sp}}{W_p + 2W_s + 1/\tau_{sp}}$$

for gain we need

$$W_p > \frac{1}{\tau_{sp}}$$

$$\frac{\sigma_a P_p}{hfA} > \frac{1}{0.001 \times 10^{-3}}$$

$$P_p > \frac{hcA}{\sigma_a \lambda (0.001 \times 10^{-3})}$$

The pump wavelength needs to be lower than 980nm. Let's use $\lambda_p = 800nm$

$$P_p > \frac{(6.6 \times 10^{-34})(3 \times 10^{8})\ 50 \times 10^{-12}}{4 \times 10^{-26}\ (0.8 \times 10^{-6})\ 0.001 \times 10^{-3}}$$

$P_p > 309W$ which is too high to be practical

2. What is the minimum pump power required for gain for the following EDFAs?
   a. Silicate L22 pumped at a wavelength of 980nm.
   b. Fluorozirconate F88 pumped at a wavelength 1480nm.

$$W_p > 1/\tau_{sp}$$

$$\frac{\sigma_a P_p}{hf A} > \frac{1}{\tau_{sp}}$$

$$P_p > \frac{hcA}{\sigma_a \lambda \tau_{sp}}$$

(a) Silicate L22 at $\lambda_p = 980nm$

$$P_p > \frac{(6.6\times10^{-34})(3\times10^{8})(50\times10^{-12})}{(10\times10^{-26})(14.5\times10^{-3})(0.98\times10^{-6})}$$

$$\boxed{P_p > 6.97\ mW}$$

(b)

$$P_p > \frac{(6.6\times10^{-34})(3\times10^{8})(50\times10^{-12})}{(2\times10^{-25})(1.48\times10^{-6})(9.4\times10^{-3})}$$

$$P_p > 3.6\ mW$$

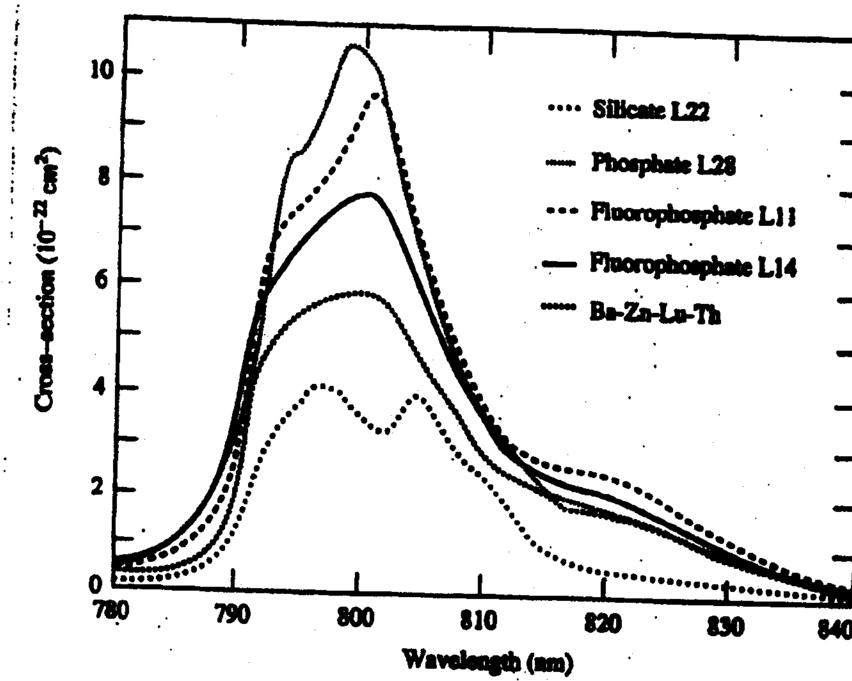

Figure 17.13 Absorption cross section of Er^{3+} at 800 nm.

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2a

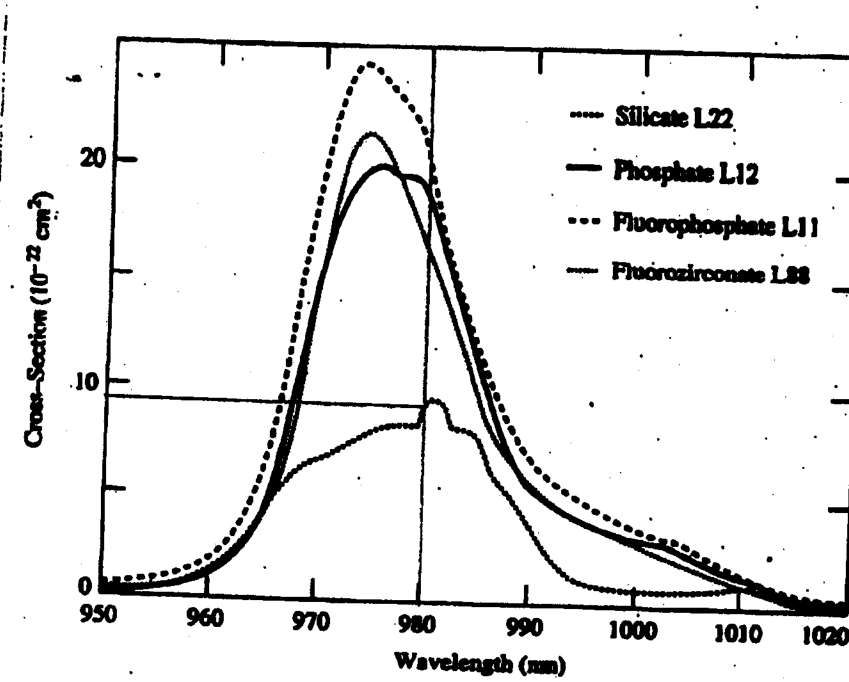

Figure 17.14 Absorption cross section of Er^{3+} at 980 nm.

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36 2b

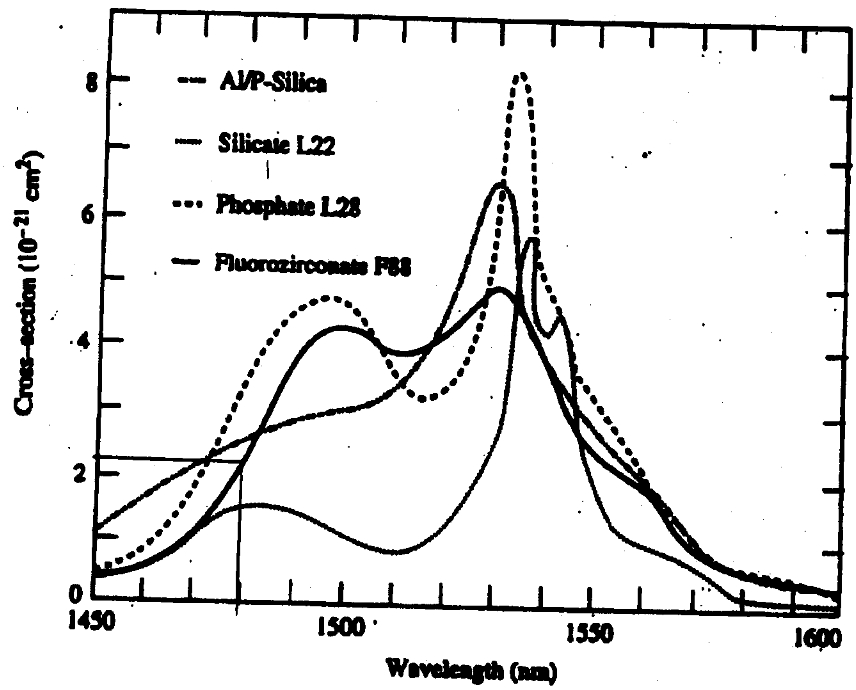

Figure 17.15 Absorption cross section of Er^{3+} at 1450 nm.

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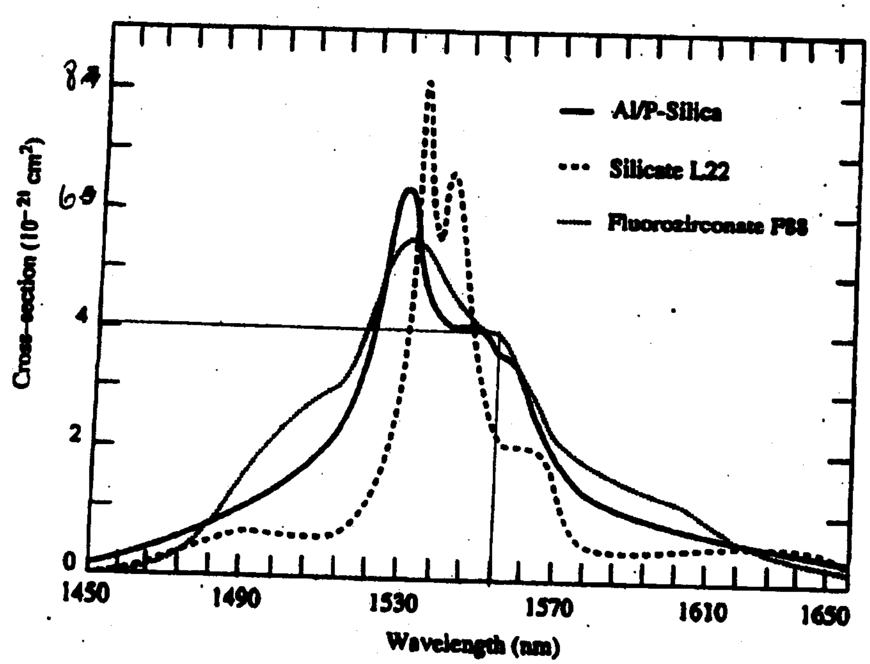

Figure 17.16 Emission cross section of Er^{3+} at 1540 nm.

Source: Reprinted, by permission, from Miniscalco, "Erbium-Doped Glasses for Fiber

3. A Fluorozironcate F88 EDFA is pumped at a wavelength of λp=1480nm, with a pump power equal to 5 times the minimum pump power required for gain. If the amplifier has a length of L=5m, what is the total gain? Assume the signal wavelength is λ_s=1554nm.

$\lambda_p = 1480nm$
$W_p = 5/\tau_{sp}$
$L = 5m$
$\lambda_s = 1554nm$
$N_t = 8 \times 10^{18}\ cm^{-3}$
$\sigma_e = 4 \times 10^{-21}$

$$g^* = (4 \times 10^{-21})\ cm^2\ (8 \times 10^{18})\ cm^{-3}$$
$$g^* = 0.032\ cm^{-1} = 3.2\ m^{-1}$$

$$g = g^* \left(\frac{W_p - 1/\tau_{sp}}{W_p + 1/\tau_{sp}} \right)$$

$$g = 3.2\ \frac{5/\tau_{sp} - 1/\tau_{sp}}{5/\tau_{sp} + 1/\tau_{sp}} =$$

$$= 3.2 \left(\frac{4}{6} \right)$$

$$g = 2.13\ m^{-1}$$

$$= (4.34)(2.13)$$
$$= 9.26\ dB/m$$

$$G = (5)(9.26)$$

$$\boxed{G = 46.3\ dB}$$

4

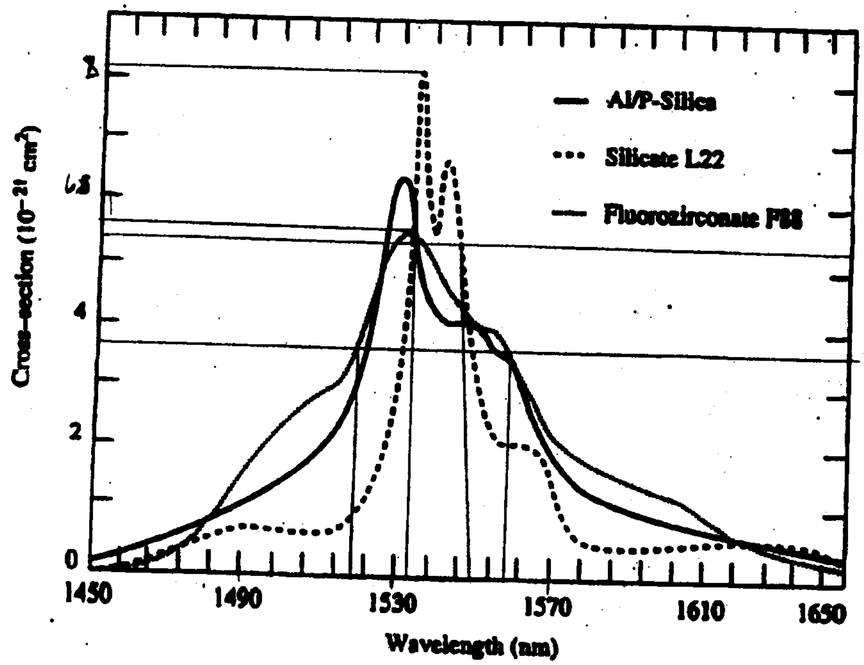

Figure 17.16 Emission cross section of Er^{3+} at 1540 nm.

Source: Reprinted, by permission, from Miniscalco. "Erbium-Doped Glasses for Fiber

4. An amplifier is constructed with a peak gain of G=30dB. Let's define the amplifier wavelength range as the wavelength range over which the normalized gain variation is less than 10dB.
   a. Which of the amplifier materials (Silicate L22 or Fluorozirconate F88) has the largest wavelength range?
   b. What is the wavelength range for the best material?

$G = 30dB$

$\Delta G = 10dB$

$$g = \sigma_e N_t \left(\frac{W_p - 1/\tau_{sp}}{W_p + 1/\tau_{sp}} \right)$$

$$g_{dB} = 4.34\, \sigma_e N_t \left(\frac{W_p - 1/\tau_{sp}}{W_p + 1/\tau_{sp}} \right)$$

$$G = 4.34\, \sigma_e N_t L \frac{W_p - 1/\tau_{sp}}{W_p + 1/\tau_{sp}}$$

$$= \left[4.34\, N_t L \left(\frac{W_p - \frac{1}{\tau_{sp}}}{W_p + 1/\tau_{sp}} \right) \right] \sigma_e$$

$$G = A_o \sigma_e$$

$$30 = A_o \sigma_{e,max}$$

$$20 = A_o \sigma_{e,min}$$

$$\frac{\sigma_{e,min}}{\sigma_{e,max}} = \frac{20}{30}$$

	L22	F88
$\sigma_{e,max}$	8.1×10^{-21}	5.6×10^{-21}
$\sigma_{e,min}$ $(\frac{2}{3}\sigma_{emax})$	5.4×10^{-21}	3.73×10^{-21}
λ_{min}	1534 nm	1518 nm
λ_{max}	1550 nm	1558 nm
$\Delta\lambda$	16 nm	40 nm

(a) Fluorozirconate F88

(b) $\Delta\lambda = 40nm$

6

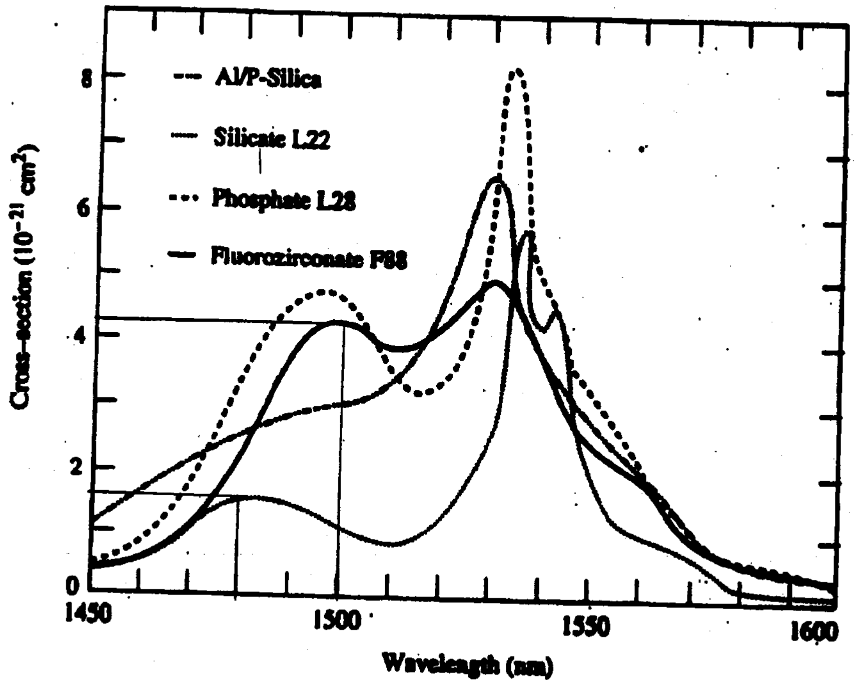

Figure 17.15 Absorption cross section of Er^{3+} at 1450 nm.

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6

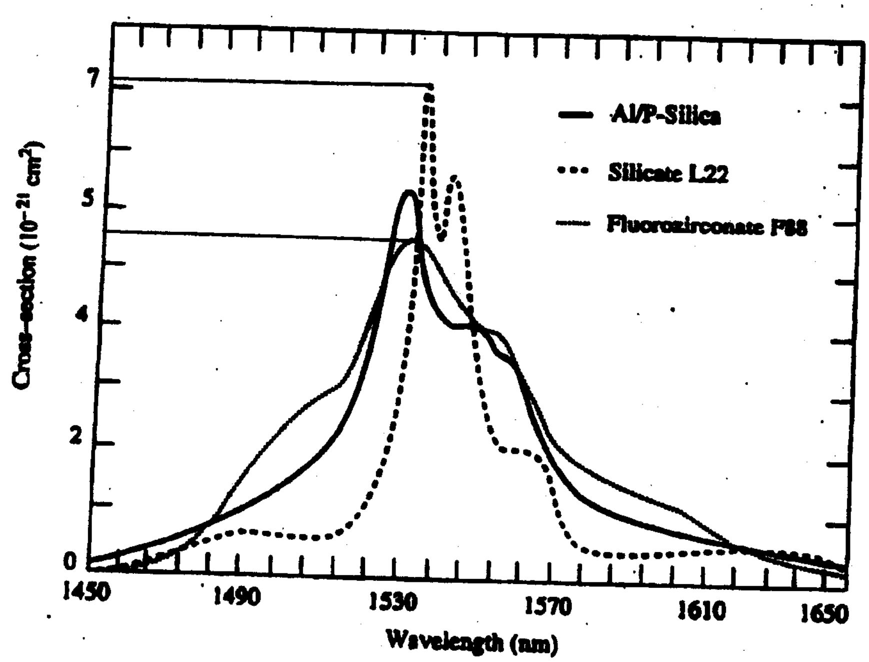

Figure 17.16 Emission cross section of Er^{3+} at 1540 nm.

Source: Reprinted, by permission, from Miniscalco, "Erbium-Doped Glasses for Fiber

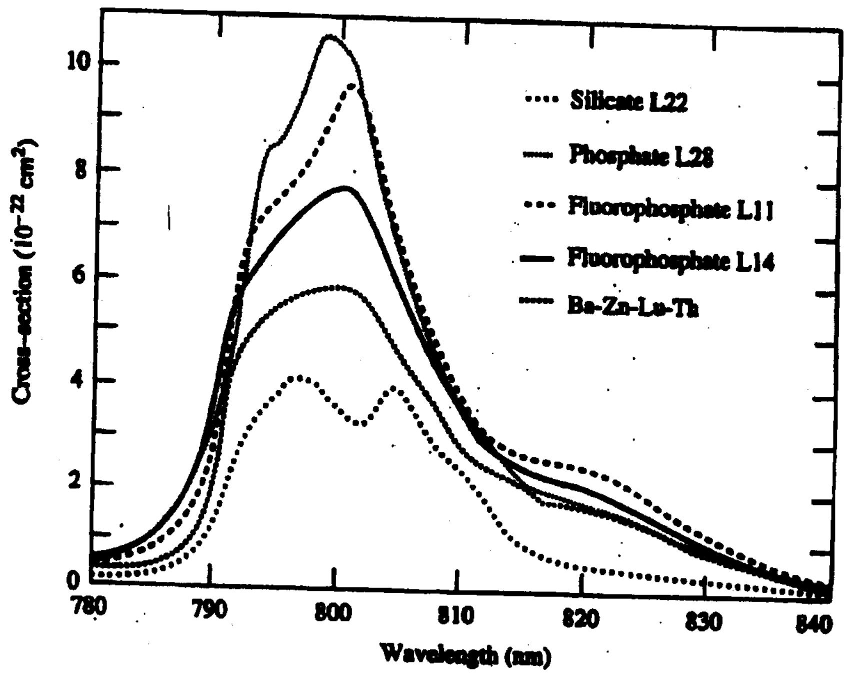

Figure 17.13 Absorption cross section of Er^{3+} at 800 nm.

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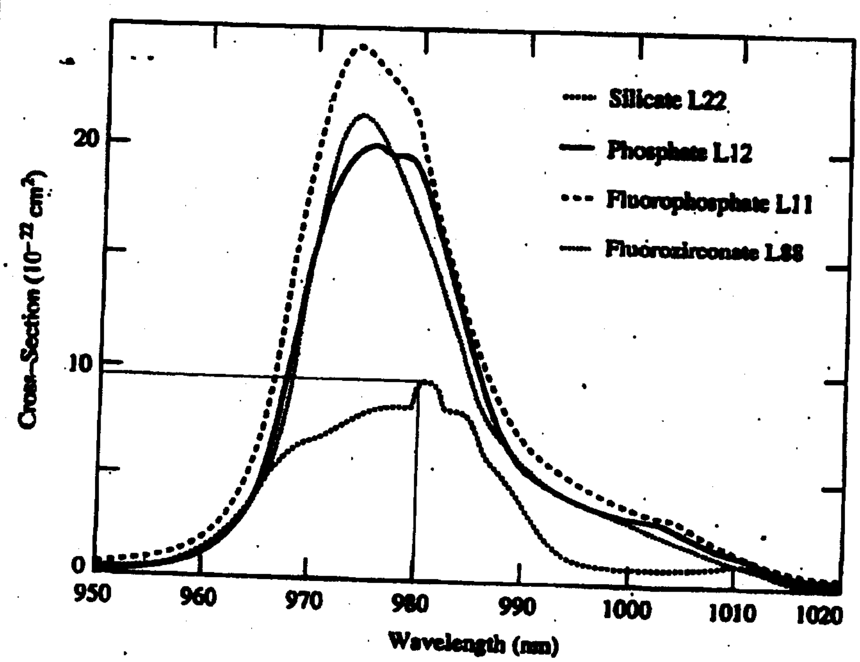

Figure 17.14 Absorption cross section of Er^{3+} at 980 nm.

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5

5 ~~6~~. **The pumping efficiency is defined as the gain divided by the pumping power in units of dB/mw.**

a. **What is the EDFA material, pump wavelength, and amplified wavelength that produce the highest pumping efficiency?**

b. **What is the pumping efficiency for (a) if the pump power is 5 times the minimum pump power for gain and the amplifier length is L=5m.**

(a) $g = \sigma_e N_T \dfrac{W_p - 1/\tau_{sp}}{W_p + 1/\tau_{sp}}$

$$G = \sigma_e N_T \frac{W_p - 1/\tau_{sp}}{W_p + 1/\tau_{sp}} (4.34)(L)$$

Assume $W_p = m/\tau_{sp}$

$$G = \sigma_e N_T \left(\frac{m-1}{m+1}\right)(4.34)(L)$$

$$W_p = \frac{m}{\tau_{sp}} = \frac{\sigma_a P_p}{h f_p A}$$

$$P_p = \frac{m h c A}{\sigma_a \tau_{sp} \lambda_p}$$

Pumping efficiency $\dfrac{G}{P_p} = \sigma_e N_T \dfrac{(m-1)}{(m+1)} (4.34)(L) \dfrac{\sigma_a \tau_{sp} \lambda_p}{m h c A}$

$$= \underbrace{\left[\sigma_e \sigma_a \tau_{sp} \lambda_p\right]}_{\text{maximize this}} N_T \left(\frac{m-1}{m+1}\right) \frac{(4.34)(L)}{m h c A}$$

Find best pump wavelength

For silicate

λ_p	σ_a	$\lambda_p \sigma_a$	
1480nm	1.5×10^{-21} cm^2	2.2×10^{-31} m^3	* best choice
980nm	10×10^{-22} cm^2	9.8×10^{-32} m^3	

For Fluorozirconate F88

λ_p	σ_a	$\lambda_p \sigma_a$	
1500nm	4.25×10^{-21}	6.4×10^{-31} m^3	* best choice
975nm	22×10^{-22}	2.1×10^{-31} m^3	

material	σ_e	τ_{sp}	$\sigma_a \lambda_p$		
Silicate	8.1×10^{-21}	14.5	2.2×10^{-31}	2.6×10^{-57} m^3/s	
Fluoro.	5.5×10^{-21}	9.4	6.4×10^{-31}	3.3×10^{-57} m^3/s	* best choice

Best design: Fluorozirconate F88, $\lambda_p = 1500$nm, $\lambda_s = 1534$nm

(b) Pump efficiency $= \dfrac{G}{P_p} = \left(3.3\times10^{-57} \dfrac{m^3}{s}\right)\left(\dfrac{4}{6}\right)(4.34)(5m) \dfrac{8.18\times10^{24}}{m^3} \Big/ 5 \,(6.6\times10^{-34})(3\times10^{8})(50\times10^{-12})$

$$\frac{7889\,dB}{W} \times \frac{1W}{10^3 mW} = \boxed{7.9\ dB/mW}$$

6. What is the gain of a Silicate L22 amplifier if $\lambda_p = 980nm$, $\lambda_s = 1550nm$, $P_p = 10mW$, $L = 5m$, $P_s = 100\mu W$?

$\sigma_e = 3 \times 10^{-21}\ cm^2$

$\sigma_a = 9 \times 10^{-22}\ cm^2$

$$W_p = \frac{\sigma_a P_p \lambda_p}{h\,c\,A} = \frac{(9\times10^{-26})(10\times10^{-3})(980\times10^{-9})}{(6.6\times10^{-34})(3\times10^{8})(50\times10^{-12})} = 89.1$$

$$W_s = \frac{\sigma_e P_{in} \lambda_s}{h\,c\,A} = \frac{(3\times10^{-25})(100\times10^{-6})(1550\times10^{-9})}{(6.6\times10^{-34})(3\times10^{8})(50\times10^{-12})} = 4.70$$

$$g = \sigma_e N_T \left(\frac{W_p - 1/\tau_{sp}}{W_p + 2W_s + 1/\tau_{sp}}\right) = (3\times10^{-25})(8\times10^{24}) \left(\frac{89.1 - 69.0}{89.1 + 9.4 + 69.0}\right)$$

$g = 0.2884\ m^{-1}$

$g_{dB} = 1.25\ dB/m$

$G_{dB} = (1.25)(5)$

$\boxed{G_{dB} = 6.26\ dB}$

7. An optical receiver has the following specifications: R=0.9 A/W, Pmin=-20dBm with B=10Gbps and BER=10^{-12}. The optical fiber has an attenuation of α=0.2 dB/km (this attenuation includes all splice losses), the transmitted optical power minus all other losses (coupling, loss margin, etc.) is 20mW, and the wavelength is λ=1550nm. Ignore any dispersion limits in this problem.
    a. What is the maximum link length?
    b. What is the maximum link length if an EDFA is used as a pre-amplifier (right before the receiver)? The EDFA has the following specifications G=30dB, NF=7dB.
    c. What is the maximum link length if an EDFA is used near the middle of the optical communications link? The EDFA has the following specifications G=30dB and NF=7dB.
    d. What is the maximum link length if 4 EDFAs are used? The EDFA has the following specifications G=30dB and NF=7dB.

(a) $P_{min} = -20\ dBm$

$P_t = 20mW = 13\ dBm$

$$L = \frac{13+20}{0.2}$$

$$\boxed{L = 165\,Km}$$

(b) The best case is $L = \dfrac{P_t - P_{min} + G}{\alpha} = \dfrac{13+20+30}{0.2} = 315$

$$\sigma^2_{ASE} = 4R^2G^2 P_{in}\, n_{sp}X\, hf\, \Delta f$$

we need $n_{sp}X$

$$NF = \left(\frac{SNR_{in}}{SNR_{out}}\right) = \left(\frac{SNR_{shot}}{SNR_{ASE}}\right) = \frac{(R\,\cancel{P_{in}})^2}{(2q\,R\,\cancel{P_{in}}\,\cancel{\Delta f})} \quad \frac{4R^{\cancel{2}}\cancel{G^2}\,\cancel{P_{in}}\,n_{sp}X\,hf\,\cancel{\Delta f}}{(R\,\cancel{G}\,\cancel{P_{in}})^2}$$

$$NF = \frac{4R\ n_{sp}X\ hf}{2q} = 2n_{sp}X\left(\frac{hf}{q}\right)R$$

$$= 2n_{sp}X\left(\frac{hf}{q}\right)\eta\left(\frac{q}{hf}\right)$$

$$NF = 2\,n_{sp}X\,\eta$$

$$\eta\left(\frac{q}{hf}\right) = \eta\,\frac{1.55}{1.24} = 0.9$$

$$\eta = 0.72$$

$$NF = 10^{0.7} = (n_{sp}X)(0.72)$$

$$n_{sp}X = 6.96$$

Since the power at the receiver will be high $\sigma^2_{ASE} \gg \sigma^2_{Th}$.
Also assume $\sigma^2_{sh} \ll \sigma^2_{ASE}$

$$SNR = 49.5 = \frac{(R P_{in} G)^2}{4 R^2 G^2 P_{in} n_{sp} \chi\, hf\, \Delta f}$$

$$49.5 = \frac{P_{in}}{4 n_{sp}\chi\, hf\, \Delta f}$$

$$P_{in} = (49.5)(4)(n_{sp}\chi)\frac{hc}{\lambda}\Delta f$$

$$P_{in} = (49.5)(4)(6.96)\frac{(6.6\times10^{-34})(3\times10^{8})}{1.55\times10^{-6}} \quad 10^{10}$$

$$P_{in} = 1.76\times10^{-6} = -27.5\ dBm$$

$$L = \frac{13+27.5}{0.2}$$

$$\boxed{L = 202.5\ Km}$$

check assumptions

$$\sigma^2_{ASE} = 4R^2G^2 P_{in}\, n_{sp}\chi \frac{hc}{\lambda}\Delta f$$

$$= (4)(0.9)^2(10^3)^2(1.76\times10^{-6})(6.96)\frac{(6.6\times10^{-34})(3\times10^{8})}{1.55\times10^{-6}} \quad 10^{10}$$

$$= 5.07\times10^{-8}$$

$$\sigma^2_{sh} = 2q\, R G P_{in}$$

$$= (2)(1.6\times10^{-19})(0.9)(10^3)(1.76\times10^{-6})$$

$$= 5.07\times10^{-22}$$

For σ^2_{th}

$$49.5 = \frac{(R P_{min})^2}{\sigma_{th}^2}$$

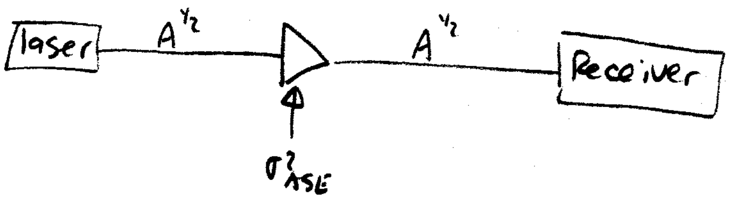
$$\sigma_{th}^2 = \frac{(0.9)^2(10^{-5})^2}{49.5} =$$

$$\sigma_{th}^2 = 1.64\times10^{-12}$$

$\sigma^2_{ASE} \gg \sigma_{th}^2$

$\sigma^2_{ASE} \gg \sigma_{sh}^2$

(C) with EDFA in the middle of the link



$$SNR = \frac{(R\,A\,G\,P_t)^2}{(\sigma^2_{ASE}\,A^{1/2}) + \sigma_{th}^2 + \sigma_{sh}^2}$$

Since σ^2_{ASE} is attenuated by $A^{1/2}$ it is probably closer to σ^2_{th}

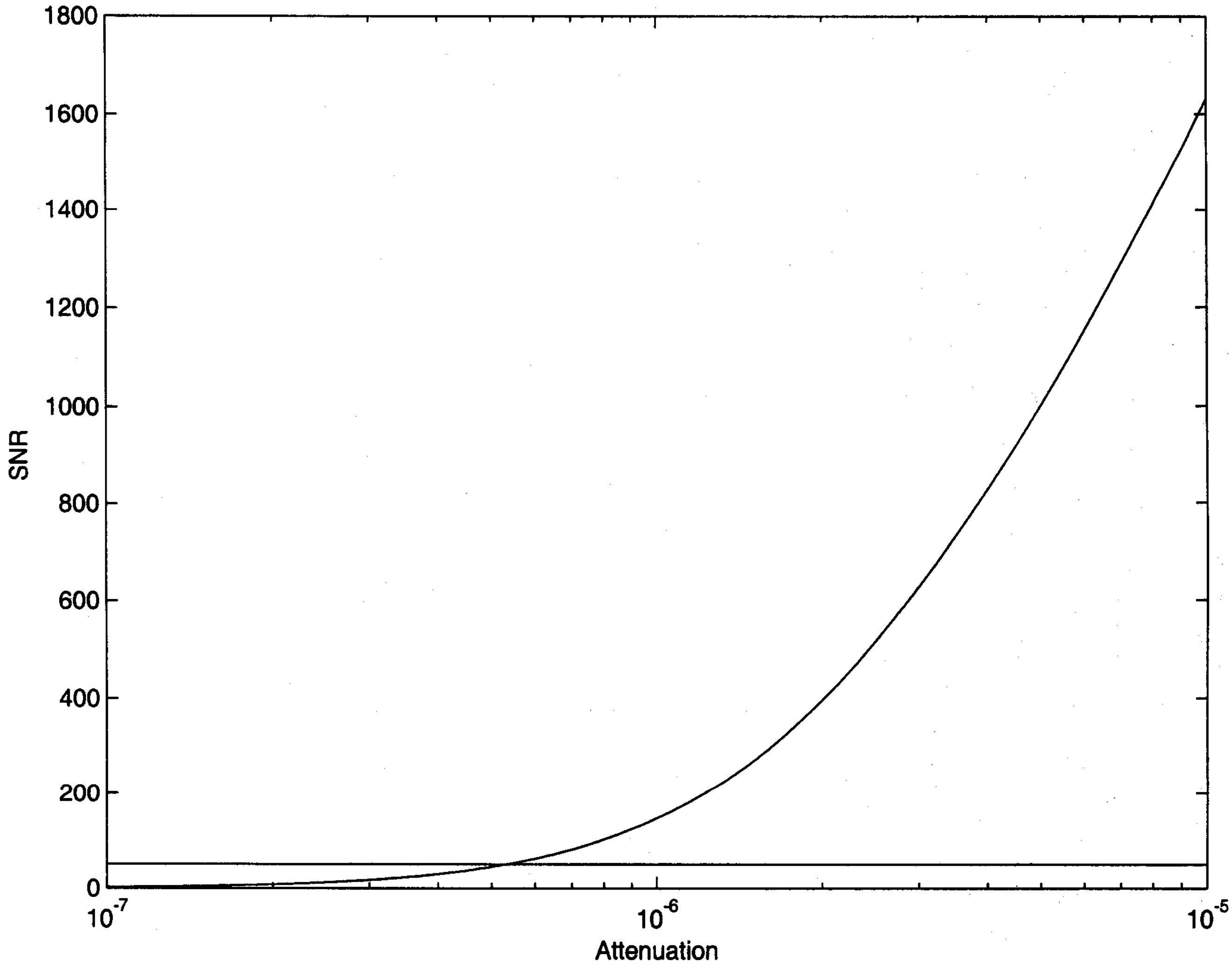
$$49.5 = \frac{(R\,A\,G\,P_t)^2}{(4R^2G^2\;A P_t\;n_{sp}\times\frac{hc}{\lambda}\Delta f\;A^{1/2}) + \sigma_{th}^2}$$

This is a nonlinear equation in terms of the attenuation A. It should be around $(300)(0.2) = 60\,dB$ so around $A = 10^{-6}$

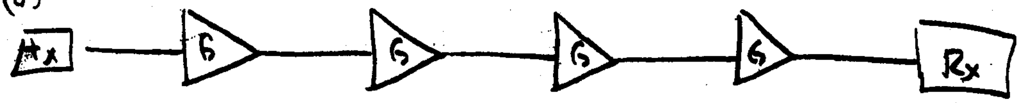
solution $A = 5.3317 \times 10^{-7}$
$= -62.73\ dB$

$$L = \frac{A}{-0.2} = \frac{-62.73}{-0.2}$$

$$\boxed{L = 313.7\ km}$$


1800
1600
1400
1200
1000
800
600
400
200
0
SNR
10^{-7}
10^{-6}
10^{-5}
Attenuation

(d)



$$I_{ph}^2 = (R\,G^4\,A\,P_t)^2$$

$$\sigma_{th}^2 = (R\,NEP)^2\,\Delta f$$

Noise from each amplifier:

$$\sigma_{ASE}^2 = 4\,(R\,P)\,(R\,G\,n_{sp}\,\chi\,hf)\,\Delta f$$

$$= 4\,R^2\,G^4 A P_t\,G\,n_{sp}\,\chi\,hf\,\Delta f$$

$$= 4\,R^2\,G^5\,A\,P_t\,n_{sp}\chi\,hf\,\Delta f$$

The noise from each amplifier is attenuated and amplified

$$\sigma_{ASE1}^2 = (4R^2 G^5 A\,P_t\,n_{sp}\chi\,hf\,\Delta f)\,A^{1/5}$$

$$\sigma_{ASE2}^2 = (4R^2 G^5 A\,P_t\,n_{sp}\chi\,hf\,\Delta f)\,G\,A^{2/5}$$

$$\sigma_{ASE3}^2 = (4R^2 G^5 A\,P_t\,n_{sp}\chi\,hf\,\Delta f)\,G^2\,A^{3/5}$$

$$\sigma_{ASE4}^2 = (4R^2 G^5 A\,P_t\,n_{sp}\chi\,hf\,\Delta f)\,G^3\,A^{4/5}$$

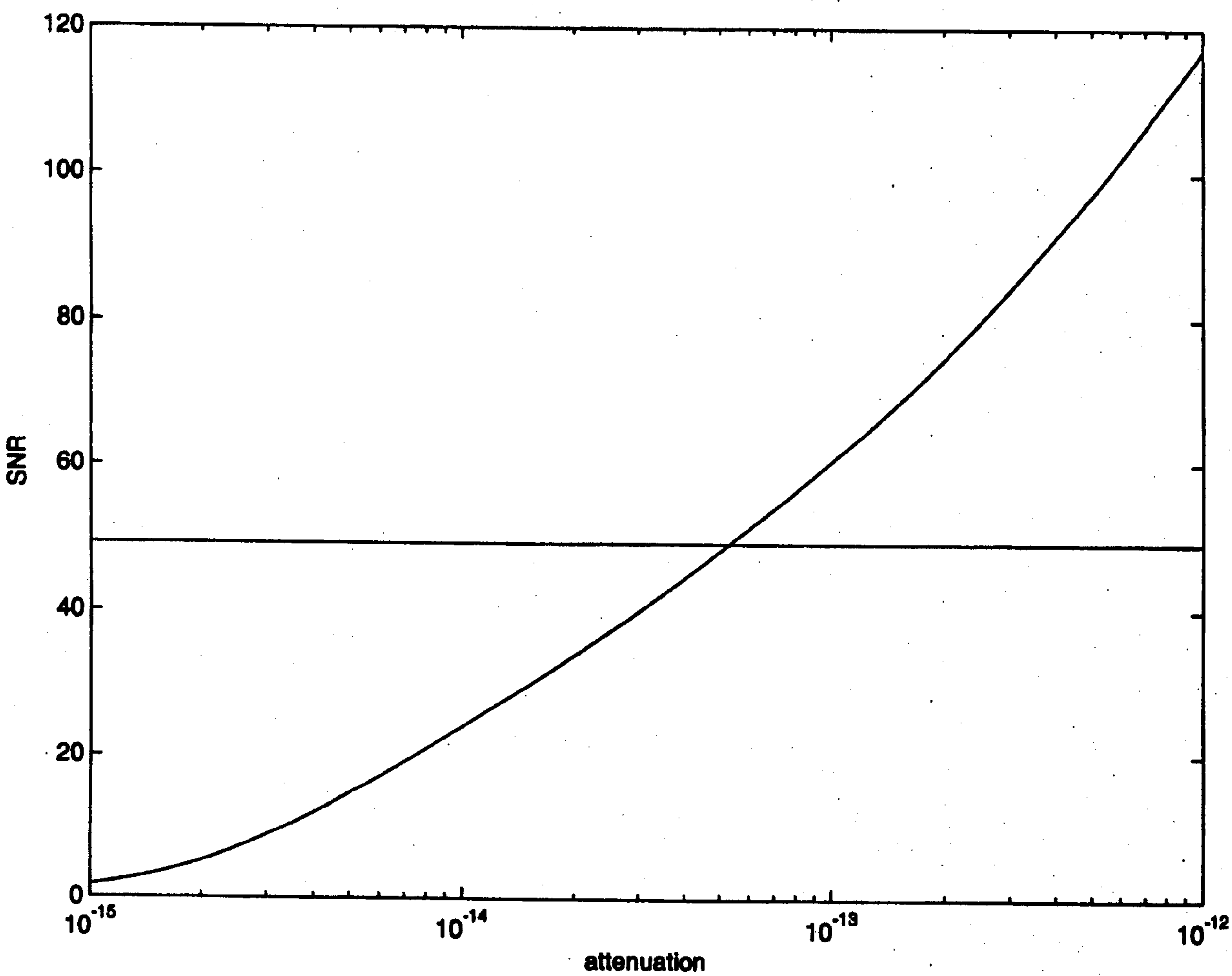
$$SNR = \frac{I_{ph}^2}{\sigma_{th}^2 + \sigma_{ASE1}^2 + \sigma_{ASE2}^2 + \sigma_{ASE3}^2 + \sigma_{ASE4}^2}$$

Plot and solve numerically to find SNR = 49

$$A = 5.27\times10^{-14}$$

$$= -132.8\text{ dB}$$

$$\boxed{L = 663.9\text{ km}}$$



120
100
80
60
40
20
0
SNR
10^{-15}
10^{-14}
10^{-13}
10^{-12}
attenuation