Homework #8

1. A free-space optical communications link is constructed with a CQF938/500 laser from JDS Uniphase, and a 264-339730-500 receiver (http://www.cmcelectronics.ca/Pdfs/CustEle_HyMic_Opti_75-Bro.pdf). Figure 1 shows the optical communications link. The laser beam is a perfect circular Gaussian beam and is spread to have a 3dB width of 10mm (this is the diameter) at the transmitter side. The receiver collects a circular diameter of 10mm. The communications link is designed to operate at B=50Mb/s and have a BER=10^{-9}. The total loss associated with all of the lenses is 3dB and there is no atmospheric distortion. What is the maximum distance between the transmitter and receiver?

![Figure 1](image)

2. (Problem 17.1) The gain ripple in units of dB is given by \( \rho = 10 \log_{10} \left( \frac{T_{\text{max}}}{T_{\text{min}}} \right) \). Show that \( \rho = 1\text{dB} \) requires \( GR = 0.0575 \).

3. (Problem 17.2)
   a. Show that the FWHM gain bandwidth of resonant transmission in a Fabry-Perot amplifier is given approximately by \( \Delta \nu_{\text{FWHM}} = \frac{c}{2n_{\text{eff}} L} \frac{1 - GR}{\pi \sqrt{GR}} \). The increase in the resonant transmission comes with an decrease of the gain bandwidth.
   b. For a SOA with \( L = 500\mu\text{m} \), \( n_{\text{eff}} = 3.6 \), \( G = 100 \), and \( R = 0.008 \), find the maximum gain at resonance and the gain bandwidth at resonance.
(1) \( P_t = 50 \text{ mW} = 17 \text{ dBm} \)

\[ P_t = 14 \text{ dBm} = 25 \text{ mW} \]

**Receiver**

- \( K = 0.17 \)
- \( \eta = 0.84 \)
- \( \text{NEP} = 34 \text{ fW/Hz} \)
- \( M = 20 \)
- \( \Delta f = 50 \text{ MHz} \)
- \( \text{BER} = 10^{-9} \)

\[
\text{SNR} = \left[ \frac{I}{28 e^{(\eta \text{NEP})}} \right]^2 = 36
\]

\[
36 = \frac{(FM P_{\text{min}})^2}{\left[ 2q M^2 F R P_{\text{min}} + R^2 \text{NEP}^2 \right] \Delta f}
\]

\[
R = 0.84 \frac{(1.55)}{1.24} = 1.05
\]

\[
F = KM + (1 + K)(2 - \frac{1}{\eta})
\]

\[
= (0.17)(20) + (1.17)(2 - 0.17)
\]

\[
F = 5.68
\]

\[
36 = \frac{(1.05)(20)(p)^2}{\left[ (8)(16x10^{-9}) (20)^2 (5.68)(1.05)(P) + (1.05)^2 \text{NEP}^2 \right] 50x10^6}
\]

\[
(1.05)(20)^2 P^2 - (36)(2)(16x10^{-9})(400)(5.68)(1.05) 50x10^6 P
\]

\[
- (36)(1.05)^2 (34x10^{-5})^2 50x10^6 = 0
\]

\[
441 P^2 - 1.374x10^{-6} P - 2.294x10^{-19} = 0
\]

\[
P = 3.117x10^{-9} = -55 \text{ dBm}
\]
The beam has an intensity of
\[ I = I_0 e^{-2 \left( \frac{\rho}{w_0} \right)^2} \]

The total power is
\[ P = 2 \pi I_0 \int_0^\infty e^{-2 \left( \frac{\rho}{w_0} \right)^2} \rho \, d\rho \]
\[ u = \frac{-\rho^2}{w_0^2} \]
\[ du = -\frac{\rho}{w_0^2} \, d\rho \]
\[ P = 2 \pi I_0 \int_0^\infty e^{u} \left( -\frac{\rho^2}{w_0^2} \right) \, du \]
\[ P = 2 \pi I_0 \frac{w_0^2}{4} \left( -e^{\frac{-\rho^2}{w_0^2}} \right)_0^\infty \]
\[ P = 2 \pi I_0 \frac{w_0^2}{4} \]
\[ I_0 = \frac{2 P}{\pi w_0^2} \]

\[ I(\rho) = \frac{2 P}{\pi w_0^2} e^{-2 \left( \frac{\rho}{w_0} \right)^2} \]

For calculating \( w_0 \)
\[ I(5\text{mm}) = \frac{I_0}{2} \]
\[ e^{-2 \left( \frac{5\text{mm}}{w_0} \right)^2} = \frac{1}{2} \]
\[ -2 \left( \frac{5}{w_0^2} \right) = 1n(\frac{1}{2}) \]
\[ w_0 = \sqrt{\frac{-50}{1n(0.5)}} \]
\[ w_0 = 8.49 \]

The beam waist expands as the beam propagates
\[ \text{Prec} = \int_0^{5\text{mm}} \int_0^{\infty} \frac{2 P}{\pi w_0^2} e^{-2 \left( \frac{\rho}{w_0} \right)^2} \rho \, d\rho \, du \]
\[ u = \frac{-\rho^2}{w_0^2} \]
\[ du = -\frac{\rho}{w_0^2} \, d\rho \]
\[ \text{Prec} = \frac{4 P}{\pi w_0^2} \int_0^{5\text{mm}} \frac{-w_0^2}{4} e^{u} \, du \]
\[ \text{Prec} = \frac{4 P}{w_0^2} \left( \frac{w_0^2}{4} \right) e^{2 \left( \frac{u}{w_0^2} \right)} \left. \right|_0^{5\text{mm}} \]
\[ \text{Prec} = P_0 \left( 1 - e^{-2 \left( \frac{5\text{mm}}{w_0^2} \right)^2} \right) \]
\[3.17 \times 10^{-9} = 25 \times 10^{-3} \left(1 - e^{-2(\xi)^2}\right)\]

\[1.247 \times 10^{-7} = 1 - e^{-2(\xi)^2}\]

\[\left(\frac{\xi}{w}\right)^2 = \ln\left(1 - 1.247 \times 10^{-7}\right)\]

\[W = \sqrt{-\frac{-50}{\ln(1 - 1.247 \times 10^{-7})}}\]

\[W = 20 \times 10^3 \text{ m} = 20 \text{ m}\]

\[Z_0 = \frac{\pi}{\gamma} W_0^2\]

\[W = W_0 \sqrt{1 + \left(\frac{L}{Z_0}\right)^2}\]

\[\left(\frac{W}{W_0}\right)^2 = 1 + \left(\frac{L}{Z_0}\right)^2\]

\[L = Z_0 \left[\left(\frac{W}{W_0}\right)^2 - 1\right]^{\frac{1}{2}}\]

\[L = \frac{\pi}{\gamma} W_0^2 \sqrt{\left(\frac{W}{W_0}\right)^2 - 1}\]

\[L = 3.45 \times 10^5 \text{ m} = 345 \text{ km}\]
The gain ripple in units of dB is given by

\[ \rho = 10 \log_{10}(T_{\text{max}} / T_{\text{min}}) \]

Show that \( \rho = 1 \) dB requires \( GR = 0.0575 \).

\[
\frac{T_{\text{max}}}{T_{\text{min}}} = \frac{(1 + G R)^2}{(1 - G R)^2}
\]

\[
\rho = 1 = 10 \log_{10} \left( \frac{(1 + G R)^2}{(1 - G R)^2} \right)
\]

\[
0.1 = 2 \log_{10} \left( \frac{1 + G R}{1 - G R} \right)
\]

\[
10^{0.05} = \left( \frac{1 + G R}{1 - G R} \right)
\]

\[
(1 - G R) \left( 10^{0.05} \right) = (1 + G R)
\]

\[
10^{0.05} - 1 = GR \left( 1 + 10^{0.05} \right)
\]

\[
GR = \frac{10^{0.05} - 1}{1 + 10^{0.05}}
\]

\[ GR = 0.0575 \]
17.2
(a) Show that the FWHM gain bandwidth of resonant transmission in a Fabry–Perot amplifier is given approximately by

\[ \Delta \nu_{\text{FWHM}} = \frac{c}{2n_{\text{eff}} L} \frac{1 - GR}{\pi \nu_{GR}} \]

The increase in the resonant transmission comes with a decrease of the gain bandwidth.

(b) For a SOA with \( L = 500 \mu \text{m} \), \( n_{\text{eff}} = 3.6 \), \( G = 100 \), and \( R = 0.008 \), find the maximum gain at resonance and the gain bandwidth at resonance.

\[ T = \frac{(1-R)^2 G}{(1-GR)^2 + 4GR \sin^2 \left( \frac{2\pi \nu_{\text{eff}} L}{c} \right)} \]

Peak occurs when \( \frac{2\pi \nu_{\text{eff}} L}{c} = m\pi \)

3 dB point occurs when

\[ (1-GR)^2 = 4GR \sin^2 \left( \frac{2\pi \nu_{\text{eff}} L}{c} (v+Dv) \right) \]

\[ \frac{1-GR}{2GR} = \sin \left[ \frac{2\pi \nu_{\text{eff}} L}{c} (v+Dv) \right] \]

\[ = \sin \left[ m\pi + \frac{2\pi \nu_{\text{eff}} L}{c} Dv \right] \]

\[ = \sin m\pi \cos \left( \frac{2\pi \nu_{\text{eff}} L}{c} Dv \right) + \cos m\pi \sin \left( \frac{2\pi \nu_{\text{eff}} L}{c} Dv \right) \]

\[ \frac{1-GR}{2GR} = \sin \left( \frac{2\pi \nu_{\text{eff}} L}{c} Dv \right) \]

Since \( \frac{2\pi \nu_{\text{eff}} L}{c} Dv \ll 1 \)

\[ \Delta \nu = \frac{1-GR}{2 \sqrt{GR}} \frac{c}{2\pi \nu_{\text{eff}} L} \]

\[ \Delta \nu_{\text{FWHM}} = 2 \Delta \nu = \frac{1-GR}{\pi GR} \frac{c}{2 \nu_{\text{eff}} L} \]

(b) \( L = 500 \mu \text{m} \), \( G = 100 \), \( n_{\text{eff}} = 3.6 \), \( R = 0.008 \)

\[ \Delta \nu_{\text{FWHM}} = 5.93 \text{ GHz} \]