Homework #8

1. A free-space optical communications link is constructed with a CQF938/500 laser from JDS Uniphase, and a 264-339730-500 receiver (http://www.cmcelectronics.ca/Pdfs/CustEle_HyMic_Opti_75-Bro.pdf). Figure 1 shows the optical communications link. The laser beam is a perfect circular Gaussian beam and is spread to have a 3dB width of 10mm (this is the diameter) at the transmitter side. The receiver collects a circular diameter of 10mm. The communications link is designed to operate at B=50Mb/s and have a BER=10^{-9}. The total loss associated with all of the lenses is 3dB and there is no atmospheric distortion. What is the maximum distance between the transmitter and receiver?

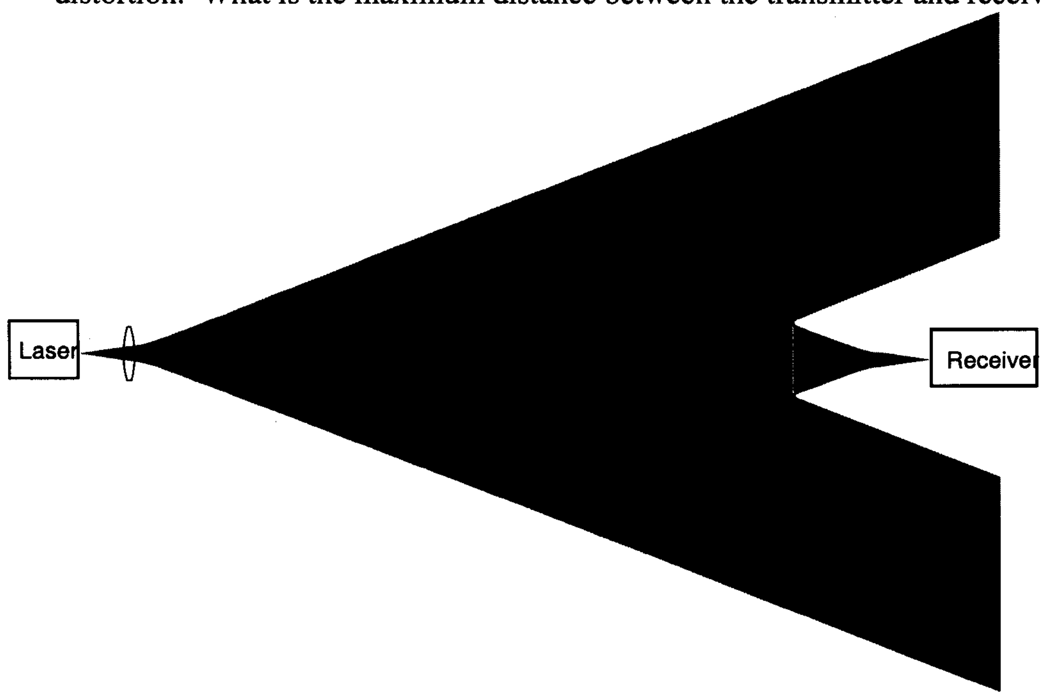

Figure 1

2. (Problem 17.1) The gain ripple in units of dB is given by $\rho = 10\log_{10}\left(\frac{T_{\max}}{T_{\min}}\right)$. Show that ρ=1dB requires GR=0.0575.

3. (Problem 17.2)
   a. Show that the FWHM gain bandwidth of resonant transmission in a Fabry-Perot amplifier is given approximately by $\Delta\nu_{FWHM} = \frac{c}{2n_{eff}L}\frac{1-GR}{\pi\sqrt{GR}}$. The increase in the resonant transmission comes with an decrease of the gain bandwidth.
   b. For a SOA with L=500μm, n_{eff}=3.6, G=100, and R=0.008, find the maximum gain at resonance and the gain bandwidth at resonance.

(1) $P_t = 50\,mW = 17\,dBm$

losses $3\,dB$

$P_t = 14\,dBm = 25\,mW$

Receiver

$K = 0.17$ $\qquad \eta = 0.84$

$NEP = 34\,fW/\sqrt{Hz}$

$M = 20$

$\Delta f = 50\,MHz$

$BER = 10^{-9}$

$$SNR = \left[\sqrt{2}\ \mathrm{erfc}^{-1}(2BER)\right]^2$$
$$= 36$$

$$36 = \frac{(R M P_{min})^2}{\left[2q M^2 F R P_{min} + R^2 NEP^2\right] \Delta f}$$

$$R = 0.84\,\frac{(1.55)}{1.24} = 1.05$$

$$F = KM + (1+K)\left(2 - \frac{1}{M}\right)$$
$$= (0.17)(20) + (1.17)\left(2 - \frac{1}{20}\right)$$

$$F = 5.68$$

$$36 = \frac{\left[(1.05)(20)(P)\right]^2}{\left[(2)(1.6\times10^{-19})(20)^2(5.68)(1.05)(P) + (1.05)^2 NEP^2\right] 50\times10^6}$$

$$(1.05)^2 (20)^2 P^2 - (36)(2)(1.6\times10^{-19})(400)(5.68)(1.05)\,50\times10^6\,P$$
$$- (36)(1.05)^2(34\times10^{-15})^2\,50\times10^6 = 0$$

$$441 P^2 - 1.374\times10^{-6} P - 2.294\times10^{-18} = 0$$

$$P = 3.117\times10^{-9} = -55\,dBm$$

The beam has an intensity of $I = I_0 \, e^{-2\left(\frac{\rho}{W_0}\right)^2}$

The total power is $P_0 = \int_0^{2\pi}\int_0^{\infty} I \rho \, d\rho \, d\phi$

$$P_0 = 2\pi I_0 \int_0^{\infty} e^{-2\frac{\rho^2}{W_0^2}} \rho \, d\rho \qquad u = \frac{-2\rho^2}{W_0^2}$$

$$du = -4\frac{\rho}{W_0^2} \, d\rho$$

$$= 2\pi I_0 \int_0^{\infty} e^{u}\left(\frac{-W_0^2}{4}\right) du$$

$$= 2\pi I_0 \frac{W_0^2}{4} \left(-e^{-2\frac{\rho^2}{W_0^2}} \Big|_0^{\infty}\right)$$

$$P_0 = 2\pi I_0 \frac{W_0^2}{4} = \frac{\pi I_0 W_0^2}{2}$$

$$I_0 = \frac{2 P_0}{\pi W_0^2}$$

$$I(\rho) = \frac{2 P_0}{\pi W_0^2} e^{-2\frac{\rho^2}{W_0^2}}$$

For calculating W_0 $\quad I(5mm) = \frac{I_0}{2}$

$$e^{-2\left(\frac{5mm}{W_0}\right)^2} = \frac{1}{2}$$

$$-2\left(\frac{25}{W_0^2}\right) = \ln\left(\tfrac{1}{2}\right)$$

$$W_0 = \sqrt{\frac{-50}{\ln(0.5)}}$$

$$W_0 = 8.49$$

The beam waist expands as the beam propagates

$$P_{rec} = \int_0^{2\pi}\int_0^{5mm} \frac{2P_0}{\pi W^2} e^{-2\left(\frac{\rho}{W}\right)^2} \rho \, d\rho \, d\phi \qquad u = -2\frac{\rho^2}{W^2}$$

$$du = -\frac{4\rho}{W^2} \, d\rho$$

$$= \frac{4\pi P_0}{\pi W^2} \int_0^{5mm} -\frac{W^2}{4} e^{u} du$$

$$= \frac{4 P_0}{W^2}\left(\frac{-W^2}{4}\right) e^{-2\left(\frac{\rho}{W}\right)^2} \Big|_0^{5mm}$$

$$= P_0\left(1 - e^{-2\left(\frac{5}{W}\right)^2}\right)$$

$$3.117\times10^{-9} = 25\times10^{-3}\left(1 - e^{-2\left(\frac{5}{w}\right)^2}\right)$$

$$1.247\times10^{-7} = 1 - e^{-2\left(\frac{5}{w}\right)^2}$$

$$-2\left(\frac{5}{w}\right)^2 = \ln\left(1 - 1.247\times10^{-7}\right)$$

$$W = \sqrt{\frac{-50}{\ln(1-1.247\times10^{-7})}}$$

$$W = 20\times10^{3}\ mm = 20\ m$$

$$Z_0 = \frac{\pi}{\lambda} W_0^2$$

$$W = W_0\sqrt{1 + \left(\frac{L}{Z_0}\right)^2}$$

$$\left(\frac{W}{W_0}\right)^2 = 1 + \left(\frac{L}{Z_0}\right)^2$$

$$L = Z_0\left[\left(\frac{W}{W_0}\right)^2 - 1\right]^{1/2}$$

$$L = \frac{\pi}{\lambda} W_0^2\sqrt{\left(\frac{W}{W_0}\right)^2 - 1}$$

$$L = 3.45\times10^{5}\ m = \boxed{345\,km}$$

17.1 The gain ripple in units of dB is given by

$$\rho = 10 \log_{10}(T_{max}/T_{min})$$

Show that $\rho = 1$ dB requires $GR = 0.0575$.

$$\frac{T_{max}}{T_{min}} = \frac{(1+GR)^2}{(1-GR)^2}$$

$$\rho = 1 = 10\log_{10} \frac{(1+GR)^2}{(1-GR)^2}$$

$$0.1 = 2\log_{10}\left(\frac{1+GR}{1-GR}\right)$$

$$10^{0.05} = \left(\frac{1+GR}{1-GR}\right)$$

$$(1-GR)\,(10^{0.05}) = (1+GR)$$

$$10^{0.05} - 1 = GR(1 + 10^{0.05})$$

$$GR = \frac{10^{0.05} - 1}{1 + 10^{0.05}}$$

$$GR = 0.0575$$

17.2

(a) Show that the FWHM gain bandwidth of resonant transmission in a Fabry–Perot amplifier is given approximately by

$$\Delta\nu_{\mathrm{FWHM}} = \frac{c}{2n_{\mathrm{eff}}L}\frac{1-GR}{\pi\sqrt{GR}}$$

The increase in the resonant transmission comes with a decrease of the gain bandwidth.

(b) For a SOA with $L = 500\ \mu\mathrm{m}$, $n_{\mathrm{eff}} = 3.6$, $G = 100$, and $R = 0.008$, find the maximum gain at resonance and the gain bandwidth at resonance.

$$T = \frac{(1-R)^2 G}{(1-GR)^2 + 4GR\sin^2\left(\frac{2\pi\nu n_{eff}L}{c}\right)}$$

peak occurs when $\frac{2\pi\nu n_{eff}L}{c} = m\pi$ $\quad \nu =$

3 dB point occurs when

$$(1-GR)^2 = 4GR\sin^2\left(\frac{2\pi n_{eff}L}{c}(\nu+\Delta\nu)\right)$$

$$\frac{1-GR}{2\sqrt{GR}} = \sin\left[\frac{2\pi n_{eff}L}{c}(\nu+\Delta\nu)\right]$$

$$= \sin\left[m\pi + \frac{2\pi n_{eff}L}{c}\Delta\nu\right]$$

$$= \cancel{\sin m\pi}^{\,0}\cos\left(\frac{2\pi n_{eff}L\Delta\nu}{c}\right) + \cancel{\cos(m\pi)}^{\,1}\sin\left(\frac{2\pi n_{eff}L}{c}\Delta\nu\right)$$

$$\frac{1-GR}{2\sqrt{GR}} = \sin\left(\frac{2\pi n_{eff}L\Delta\nu}{c}\right) \qquad \text{since } \left(\frac{2\pi n_{eff}L}{c}\Delta\nu\right) \ll 1$$

$$\frac{1-GR}{2\sqrt{GR}} = \frac{2\pi n_{eff}L\Delta\nu}{c}$$

$$\Delta\nu = \frac{1-GR}{2\sqrt{GR}}\frac{c}{2\pi n_{eff}L}$$

$$\Delta\nu_{FWHM} = 2\Delta\nu = \boxed{\frac{1-GR}{\pi\sqrt{GR}}\frac{c}{2n_{eff}L}}$$

(b) $L = 500\,mm$ $\quad G = 100$

$n_{eff} = 3.6$ $\quad R = 0.008$

$$\boxed{\Delta\nu_{FWHM} = 5.93\ GHz}$$