

Homework 6

1. If an electric field E_0 is applied in the $[0,1,0]$ direction inside ADP, find the principal axes at $E_0=10^4$ V/m (Do not assume that if it is small it is equal to zero.). Use $\lambda=633\text{nm}$. What is the propagation direction for maximum birefringence and the value of the birefringence?
2. A bulk cube of KD*P is placed in one arm of a Mach-Zehnder interferometer to create a modulator. To couple light through the cube the width of the cube must be greater than 10mm . Use a wavelength of $\lambda=546\text{nm}$.
 - a. Design a modulator using the KD*P cube to have the lowest voltage for 100% extinction. What is the required voltage?
 - b. An integrated MZ modulator is fabricated from the same material. The length is still $L=10\text{mm}$ but the thickness is now $d=10\mu\text{m}$. Design the modulator and calculate the voltage for 100% extinction. (An integrated modulator has to operate with the voltage applied perpendicular to the propagation direction.)
3. A bulk modulator is designed to modulate a CO_2 laser, which has a wavelength of $\lambda=10.6\mu\text{m}$. The modulator is designed to use polarization filtering and uses a cube of material with dimension of 10mm . Design the modulator and determine the best cube orientation, and the required voltage range for entire extinction.
 - a. What is the best material to use? You can only use materials from Table 9.2 that are listed for a wavelength of $\lambda=10.6\mu\text{m}$.
 - b. What is the $V\pi$?

For problems 4-6 refer to the Avanex specification sheet and the reference article on lithium niobate modulators. An x-cut crystal means that the direction normal to the surface of the wafer is the x-direction of the crystal.

4. A modulator is fabricated using lithium niobate. Calculate the $V\pi$ of the modulator as a function of d/L , where d is the electrode spacing and L is the length of the modulator. Assume that the modulator is operated in a push-pull configuration. For the following cases
 - a. y-propagating using an x-cut crystal
 - b. x-propagating using a z-cut crystal
5. Why is the x-cut crystal used for the Avanex modulator?
6. Estimate the d/L ratio for the Avanex modulator.
7. An amplitude modulator is configured in a polarization filter setup. The input polarization state is given by $\vec{E} = 4e^{j0.1}\hat{x} + 2e^{j0.5}\hat{z}$ and the output is passed through a linear polarizer oriented at 45 degrees. The modulator has a length L and the two

indices of refraction are given by $n_x = n_o - \frac{n_o^3}{2} r_{13} \frac{V}{d}$ and $n_z = n_e - \frac{n_e^3}{2} r_{33} \frac{V}{d}$, where $n_o=2.286$, $n_e=2.2$, $r_{13}=9.6\text{pm/V}$, $r_{33}=30.9\text{ pm/V}$, $\lambda=1550\text{nm}$, $d=1000$, and $L=2\text{cm}$.

- a. Plot the relative optical power as a function of voltage.
- b. What is the extinction ratio (P_{\min}/P_{\max})?

1. If an electric field E_0 is applied in the $[0,1,0]$ direction inside ADP, find the principal axis. Is it dependent on the magnitude of the electric field E_0 ? If yes, find the principle axis at $E_0=10^4$ V/m. Use $\lambda=633$ nm. What is the propagation direction for maximum birefringence and the value of the birefringence?

ADP is 42m

$$\Delta n = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{11} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{33} \end{bmatrix} \begin{bmatrix} 0 \\ E_0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ r_{41} E_0 \\ 0 \end{bmatrix}$$

$$n_o = 1.5266 \quad n_e = 1.4808$$

$$\frac{1}{n_o^2} x^2 + \frac{1}{n_o^2} y^2 + \frac{1}{n_e^2} z^2 + 2r_{41} E_0 xz = \frac{1}{n^2}$$

Need a coordinate rotation about the y-axis

$$B_1 = B$$

$$A_1 = A \cos^2 \theta + C \sin^2 \theta + D \sin \theta \cos \theta$$

$$C_1 = A \sin^2 \theta + C \cos^2 \theta - D \sin \theta \cos \theta$$

$$E_1 = -2A \sin \theta \cos \theta + 2C \sin \theta \cos \theta + D(\cos^2 \theta - \sin^2 \theta)$$

$$\text{set } E_1 = 0$$

$$0 = (-A+C) 2 \sin \theta \cos \theta + D(\cos^2 \theta - \sin^2 \theta)$$

$$(A-C) \sin 2\theta = D \cos 2\theta$$

$$\tan 2\theta = \frac{D}{A-C}$$

$$\tan 2\theta = \frac{2r_{41} E_0}{\frac{1}{n_o^2} - \frac{1}{n_e^2}} =$$

$$\theta = \frac{1}{2} \tan^{-1} \left(\frac{2(23.41 \times 10^{-12})(10^4)}{\frac{1}{1.5266^2} - \frac{1}{1.4808^2}} \right)$$

$$\theta = -5 \times 10^{-9} \text{ degrees}$$

$$B_1 = \frac{1}{n_o^2}$$

$$A_1 = \frac{1}{n_o^2} - 2r_{41} E_0 (-8.6857 \times 10^{-6})$$

$$C_1 = \frac{1}{n_e^2} + 2r_{41} E_0 (-8.6857 \times 10^{-6})$$

Max birefringence when propagating in the y-direction

$$n_x = n_o + n_o^3 r_{41} E_0 \sin \theta \cos \theta$$

$$= 1.5266 - 7.23 \times 10^{-12}$$

$$n_y = n_e + n_e^3 r_{41} E_0 \sin \theta \cos \theta = 1.4808 + 6.6 \times 10^{-12}$$

$$\Delta n = (1.5266 - 7.23 \times 10^{-12}) - (1.4808 + 6.6 \times 10^{-12})$$

$$\Delta n = 0.0458 - 1.38 \times 10^{-4}$$

2. A bulk cube of KD*P is placed in one arm of a Mach-Zehnder interferometer to create a modulator. To couple light through the cube the width of the cube must be greater than 10mm. Use a wavelength of $\lambda=546\text{nm}$.

- Design a modulator using the KD*P cube to have the lowest voltage for 100% extinction. What is the required voltage?
- An integrated MZ modulator is fabricated from the same material. The length is still $L=10\text{mm}$ but the thickness is now $d=10\mu\text{m}$. Design the modulator and calculate the voltage for 100% extinction. (An integrated modulator has to operate with the voltage applied perpendicular to the propagation direction.)

(a) KD*P is 42m

$$r = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{pmatrix} \quad r_{63} > r_{41}$$

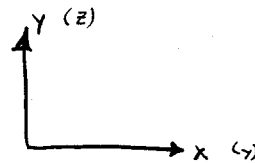
From the previous problem, if $\vec{E} = E_0 \hat{x}$ or $E_0 \hat{y}$ then $\theta \approx 0$ and there is very little modulation so choose

$$\vec{E} = E_0 \hat{z}$$

$$\frac{1}{n_0^2} x^2 + \frac{1}{n_0^2} y^2 + \frac{1}{n_0^2} z^2 + \underbrace{2r_{63} E_0}_{D} x y = \frac{1}{n^2}$$

rotate about the z-axis

y becomes x
z becomes y



$$\begin{aligned} X = x_1 &\Rightarrow Z = z_1 \\ Y = y_1 \cos \theta - z_1 \sin \theta &\Rightarrow X = x_1 \cos \theta - y_1 \sin \theta \\ Z = y_1 \sin \theta + z_1 \cos \theta &\Rightarrow Y = x_1 \sin \theta + y_1 \cos \theta \end{aligned}$$

$$A_1 = B \sin^2 \theta + A \cos^2 \theta - D \sin \theta \cos \theta$$

$$B_1 = B \cos^2 \theta + A \sin^2 \theta + D \sin \theta \cos \theta$$

$$C_1 = C$$

$$D_1 = -2B \sin \theta \cos \theta + 2A \sin \theta \cos \theta + D (\cos^2 \theta - \sin^2 \theta)$$

Set $D_1 = 0$

$$(B-A) 2 \sin \theta \cos \theta = D (\cos^2 \theta - \sin^2 \theta)$$

$$(B-A) \sin 2\theta = D \cos 2\theta$$

$$\tan 2\theta = \frac{D}{B-A} = \frac{2r_{63} E_0}{\frac{1}{n_0^2} - \frac{1}{n_0^2}} = \infty$$

$$\tan 2\theta = 90^\circ$$

$$\theta = 45^\circ$$

$$A_1 = \frac{1}{2} B + \frac{1}{2} A - \frac{1}{2} D$$

$$= \frac{1}{2} \frac{1}{n_0^2} + \frac{1}{2} \frac{1}{n_0^2} - \frac{1}{2} (2r_{63} E_0) = \frac{1}{n_0^2} - r_{63} E_0$$

$$B_1 = \frac{1}{2} (B + A + D)$$

$$= \frac{1}{2n_0^2} + \frac{1}{2n_0^2} + r_{63} E_0 = \frac{1}{n_0^2} + r_{63} E_0$$

$$C_1 = \frac{1}{n_0^2}$$

$$D_1 = 0$$

$$\frac{1}{n_x^2} = \frac{1}{n_0^2} - r_{63} E_0$$

$$n_x = \left[\frac{1}{n_0^2} - r_{63} E_0 \right]^{-1/2}$$

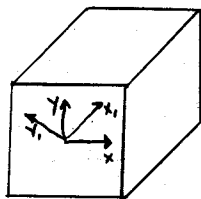
$$n_x = n_0 \left(1 - r_{63} n_0^2 E_0 \right)^{-1/2}$$

$$n_x = n_0 + \frac{1}{2} r_{63} n_0^3 E_0$$

$$n_y = n_0 - \frac{1}{2} r_{63} n_0^3 E_0$$

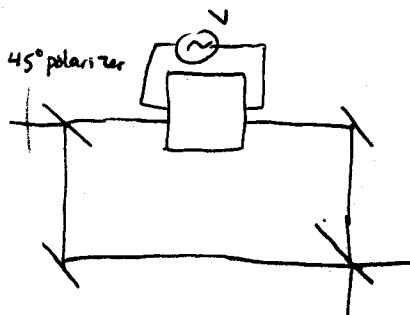
$$n_z = n_0$$

We want the electric field in either the x_1 or y_1 direction



2 options: (1) propagate in the \hat{z} direction with field in y_1 direction
(2) propagate in the \hat{x}_1 direction

For option 2 we are not normal to the surface so choose option 1



this requires either a ring electrode or a transparent electrode

The field can be in either the x_1 or y_1 direction, which is 45° from the cube axes.

for a Mach Zehnder

$$\frac{I_{out}}{I_{in}} = \frac{1}{2} (1 + \cos \Delta\phi)$$

$$\Delta\phi = \frac{2\pi}{\lambda} L \Delta n$$

$$= \frac{2\pi}{\lambda} \left(\frac{1}{2}\right) (n_{o3}) n_o^3 \sqrt{\frac{L}{d}}$$

$$V_{\pi} = \frac{\pi}{546 \times 10^{-9}} (26.8 \times 10^{-12}) (1.5079^3) \sqrt{L}$$

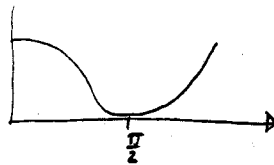
$$V_{\pi} = \frac{(546 \times 10^{-9})}{(26.8 \times 10^{-12})(1.5079^3)}$$

$$V_{\pi} = 5942 \text{ V}$$

(b) Now $\frac{d}{L} = \frac{10 \times 10^{-6}}{10 \times 10^{-3}} = 10^{-3}$

$$V_{\pi} = 5942 \times 10^{-3}$$

$$V_{\pi} = 5.94 \text{ V}$$



3. A bulk modulator is designed to modulate a CO₂ laser, which has a wavelength of $\lambda=10.6\mu\text{m}$. The modulator is designed to use polarization filtering and uses a cube of material with dimension of 10mm. Design the modulator and determine the best cube orientation, and the required voltage range for entire extinction.

- What is the best material to use? You can only use materials from Table 9.2 that are listed for a wavelength of $\lambda=10.6\mu\text{m}$.
- What is the $V\pi$?

$\lambda=10.6\mu\text{m}$
 We need to use CdTe, GaAs, ZnSe, ZnTe. These are all 43m symmetry crystals and are initially isotropic materials

$$r = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{41} \end{bmatrix}$$

$$\frac{1}{n_o^2}x + \frac{1}{n_o^2}y + \frac{1}{n_o^2}z + 2r_{41}E_x yz = \frac{1}{n^2}$$

rotate about the x-axis

$$A_1 = A$$

$$B_1 = B \cos^2\theta + C \sin^2\theta + D \cos\theta \sin\theta$$

$$C_1 = B \sin^2\theta + C \cos^2\theta - D \cos\theta \sin\theta$$

$$D_1 = -2B \sin\theta \cos\theta + 2C \sin\theta \cos\theta + D(\cos^2\theta - \sin^2\theta)$$

$$D_1 = 0 = (C-B) 2\sin\theta \cos\theta + D(\cos^2\theta - \sin^2\theta)$$

$$B-C \sin 2\theta = D \cos 2\theta$$

$$\tan 2\theta = \frac{D}{B-C} = \frac{2r_{41}E_x}{\frac{1}{n_o^2} - \frac{1}{n_o^2}} = \infty$$

$$2\theta = 90^\circ$$

$$\theta = 45^\circ$$

$$A_1 = A = \frac{1}{n_o^2}$$

$$B_1 = B \cos^2(45) + C \sin^2(45) + D \cos(45) \sin(45)$$

$$= \frac{1}{n_o^2} + 2r_{41}E_x \left(\frac{1}{2}\right) = \frac{1}{n_o^2} + r_{41}E_x$$

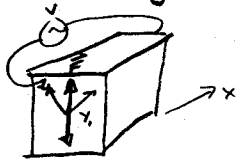
$$C_1 = \frac{1}{n_o^2} - r_{41}E_x$$

$$D_1 = 0$$

$$n_{y1} = n_o - \frac{1}{2} r_{41} n_o^3 E_x$$

$$n_{z1} = n_o + \frac{1}{2} r_{41} n_o^3 E_x$$

For polarization filtering we want to propagate in the \hat{x} -direction.



$$\phi_1 = \frac{2\pi}{\lambda} L n - \left(\frac{2\pi}{\lambda}\right) \left(\frac{1}{2}\right) r_{41} n^3 v \frac{L}{d}$$

$$\phi_2 = \frac{2\pi}{\lambda} L n + \frac{2\pi}{\lambda} \left(\frac{1}{2}\right) r_{41} n^3 v \frac{L}{d}$$

$$\Gamma = \phi_2 - \phi_1$$

$$\frac{I_{out}}{I_{in}} = \sin^2 \left(\frac{\phi_2 - \phi_1}{2} \right) = \sin^2 \left(\frac{\pi}{2} \frac{V}{V_{\pi}} \right)$$

$$\left(\frac{\pi}{2}\right) \left(\frac{V}{V_{\pi}}\right) = \frac{1}{2} \left[\frac{\pi}{\lambda} r_{41} n^3 \frac{L}{d} + \frac{\pi}{\lambda} r_{41} n^3 \frac{L}{d} \right] v$$

$$\left(\frac{\pi}{2}\right) \left(\frac{1}{V_{\pi}}\right) = \frac{\pi}{\lambda} r_{41} n^3 \frac{L}{d}$$

$$V_{\pi} = \left(\frac{\pi}{2}\right) \left(\frac{\lambda}{\pi}\right) \left(\frac{1}{r_{41} n^3}\right) \frac{d}{L}$$

$$V_{\pi} = \frac{\lambda}{2 r_{41} n^3}$$

From the table choose the largest $r_{41} n^3$

CdTe : 120

GaAs : 54

ZnSe : 35

ZnTe : 77

choose CdTe

$$V_{\pi} = \frac{10.6 \times 10^{-6}}{(2)(120 \times 10^{-12})}$$

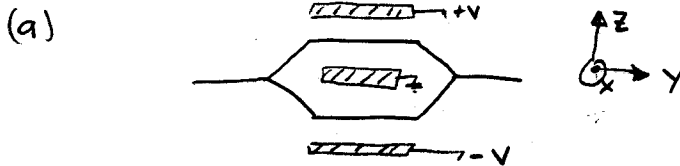
$$V_{\pi} = 44 \text{ kV}$$

For problems 4-6 refer to the Avanex specification sheet and the reference article on lithium niobate modulators. An x-cut crystal means that the direction normal to the surface of the wafer is the x-direction of the crystal.

4. A modulator is fabricated using lithium niobate. Calculate the $V\pi$ of the modulator as a function of d/L , where d is the electrode spacing and L is the length of the modulator. Assume that the modulator is operated in a push-pull configuration. For the following cases

- y-propagating using an x-cut crystal
- x-propagating using a z-cut crystal

For both cases, assume lateral applied electric field.



$$\hat{E} = E_z \hat{z}$$

$$r = \begin{bmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ E_z \end{bmatrix}$$

$$\left(\frac{1}{n_0^2} + r_{13} E_z\right) x + \left(\frac{1}{n_0^2} + r_{13} E_z\right) y + \left(\frac{1}{n_e^2} + r_{33} E_z\right) z = \frac{1}{n^2}$$

$$n_x = n_0 - \frac{1}{2} n_0^3 r_{13} E_z$$

$$n_y = n_0 - \frac{1}{2} n_0^3 r_{13} E_z$$

$$n_z = n_e - \frac{1}{2} n_e^3 r_{33} E_z$$

The best field direction is \hat{z} because $r_{33} n_e^3 > r_{13} n_0^3$

$$\phi_1 = \frac{2\pi}{\lambda} n_e L - \frac{2\pi}{\lambda} \left(\frac{1}{2}\right) n_e^3 r_{33} V \frac{L}{d}$$

$$\phi_2 = \frac{2\pi}{\lambda} n_e L + \frac{2\pi}{\lambda} \left(\frac{1}{2}\right) n_e^3 r_{33} V \frac{L}{d}$$

$$\Delta\phi = \phi_2 - \phi_1 = \frac{2\pi}{\lambda} n_e^3 r_{33} V \frac{L}{d}$$

$$\frac{I_{out}}{I_{in}} = \frac{1}{2} \left(1 + \cos\left(\frac{2\pi}{\lambda} n_e^3 r_{33} V \frac{L}{d}\right)\right)$$

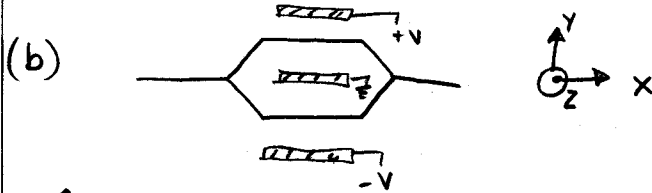
$$\frac{2\pi}{\lambda} n_e^3 r_{33} V \frac{L}{d} = \pi$$

$$V\pi = \frac{\lambda}{2} \frac{1}{n_e^3 r_{33}} \frac{d}{L}$$

for $\lambda = 1.55 \mu\text{m}$ use the $\lambda = 3.39 \mu\text{m}$ value in the table
 $r_{33} = 28 \text{ pm/V}$, $n_e = 2.073$

$$V_{\pi} = \frac{(1.55 \times 10^{-6})}{2} \frac{1}{(2.073^3)(28 \times 10^{-12})} \frac{d}{L}$$

$$V_{\pi} = 3108 \frac{d}{L}$$



$$\vec{E} = E_y \hat{y}$$

$$\left(\frac{1}{n_o^2} - r_{22} E_y\right) x + \left(\frac{1}{n_o^2} + r_{22} E_y\right) y + \frac{1}{n_e^2} z + 2r_{51} E_y yz = \frac{1}{n^2}$$

$$A_1 = A$$

$$B_1 = B \cos^2 \theta + C \sin^2 \theta + D \sin \theta \cos \theta$$

$$C_1 = B \sin^2 \theta + C \cos^2 \theta - D \sin \theta \cos \theta$$

$$D_1 = -2B \sin \theta \cos \theta + 2C \sin \theta \cos \theta + D (\cos^2 \theta - \sin^2 \theta)$$

$$D_1 = 0 = (C - B) 2 \sin \theta \cos \theta + D (\cos^2 \theta - \sin^2 \theta)$$

$$(B - C) \sin 2\theta = D \cos 2\theta$$

$$\tan 2\theta = \frac{D}{B - C} = \frac{2 r_{51} E_y}{\frac{1}{n_o^2} + r_{22} E_y - \frac{1}{n_e^2}}$$

$$\theta = \tan^{-1} \frac{2 r_{51} E_y}{\frac{1}{n_o^2} - \frac{1}{n_e^2} + r_{22} E_y}$$

$$\theta \approx 0$$

$$A_1 = A = \frac{1}{n_o^2} - r_{22} E_y$$

$$B_1 \approx B = \frac{1}{n_o^2} + r_{22} E_y$$

$$C_1 \approx C = \frac{1}{n_e^2}$$

$$D_1 = 0$$

the optical field is in either the \hat{y} or \hat{z} direction. The best case is the \hat{y} direction

$$n_y = n_o - \frac{1}{2} n_o^3 r_{22} E_y$$

$$\Delta \phi = 2 \left(\frac{2\pi}{\lambda}\right) \left(\frac{1}{2}\right) n_o^3 r_{22} E_y L = \frac{2\pi}{\lambda} n_o^3 r_{22} E_y L$$

$$\frac{I_{out}}{I_{in}} = \frac{1}{2} (1 + \cos(\frac{2\pi}{\lambda} n_o^3 r_{22} E_y L))$$

$$\pi = \frac{2\pi}{\lambda} n_o^3 r_{22} V_{\pi} \frac{L}{d}$$

$$V_{\pi} = \frac{\lambda}{2 n_o^3 r_{22}} \frac{d}{L} = \frac{1.55 \times 10^{-6}}{(4)(2.136^3)(3.1 \times 10^{-12})} \frac{d}{L} = 25652 \frac{d}{L}$$

(5) X-cut crystal is easier to create side by side electrodes
the produce a field in the z-direction.

X-cut crystal also has a reduced piezo electric effect,
which can cause charge build up which varies the
bias of the modulator

(6) For the Avanex modulator
 $\vec{E} = E_z \hat{z}$, Y-propagating, push-pull

$$V_{\pi} = 3108 \frac{d}{L}$$

$$V_{\pi} = 5V = 3108 \frac{d}{L}$$

$$\frac{d}{L} = \frac{5}{3108}$$

$$\frac{d}{L} = 0.0016$$

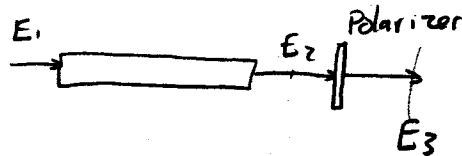
7. An amplitude modulator is configured in a polarization filter setup. The input polarization state is given by $\vec{E} = 4e^{j0.1}\hat{x} + 2e^{j0.5}\hat{z}$ and the output is passed through a linear polarizer oriented at 45 degrees. The modulator has a length L and the two

indices of refraction are given by $n_x = n_o - \frac{n_o^3}{2} r_{13} \frac{V}{d}$ and $n_z = n_e - \frac{n_e^3}{2} r_{33} \frac{V}{d}$, where $n_o=2.286$, $n_e=2.2$, $r_{13}=9.6\text{pm/V}$, $r_{33}=30.9\text{pm/V}$, $\lambda=1550\text{nm}$, $d=1000$, and $L=2\text{cm}$.

- Plot the relative optical power as a function of voltage.
- What is the extinction ratio (P_{\min}/P_{\max})?

$$\vec{E} = 4e^{j0.1}\hat{x} + 2e^{j0.5}\hat{z}$$

$$E_1 = \begin{bmatrix} 4e^{j0.1} \\ 2e^{j0.5} \end{bmatrix}$$



$$n_x = n_o - \frac{1}{2} n_o^3 r_{13} \frac{V}{d}$$

$$\phi_x = \frac{2\pi}{\lambda} n_x L - \frac{\pi}{\lambda} n_o^3 r_{13} \frac{V}{d} L$$

$$= \left(\frac{2\pi}{1.55 \times 10^6} \right) (2.286) (0.02) - \left(\frac{\pi}{1.55 \times 10^6} \right) (2.286^3) (9.6 \times 10^{-12}) V (10^3)$$

$$\phi_y = \left(\frac{2\pi}{1.55 \times 10^6} \right) (2.2) (0.02) - \left(\frac{\pi}{1.55 \times 10^6} \right) (2.2^3) (30.9 \times 10^{-12}) (10^3)$$

$$E_2 = \begin{bmatrix} 4e^{j0.1} e^{j\phi_x} \\ 2e^{j0.5} e^{j\phi_y} \end{bmatrix}$$

$$E_3 = [E_2] \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\frac{P_{\min}}{P_{\max}} = 0.33$$

```
*****
%EO modulation using polarization filtering
%with unbalanced polarizations.
*****
lambda=1.55e-6;
no=2.286;
ne=1.2;
r13=9.6e-12;
r33=30.9e-12;
L=2e-2;
d=1e-3*L;

E1=[4*exp(j*0.1);2*exp(j*0.5)]
pol=1/2*[1,1;1,1];
VV=linspace(0,60,101);
for lp=1:length(VV)
    V=VV(lp);
    phix=2*pi/lambda*no*L-pi/lambda*no^3*r13*V*L/d;
    phiz=2*pi/lambda*ne*L-pi/lambda*ne^3*r33*V*L/d;

    E2=E1.*[exp(j*phix);exp(j*phiz)];
    E3=pol*E2;
    P(lp)=norm(E3);
end

plot(VV,P./max(P))
xlabel('voltage')
ylabel('Normalized power')
```

