Homework 4

1. An optical communications link is constructed with SMF28 optical fiber (use $\lambda_0=1310\text{nm}$) and operated with a bit rate of $B=10\text{Gbps}$. What is the maximum distance using the following optical sources? (Ignore any attenuation limits.)
   a. An infrared LED with a center wavelength of $\lambda=1310\text{nm}$ and a linewidth of $\Delta\lambda=30\text{nm}$.
   b. A FP laser diode with a center wavelength of $\lambda=1310\text{nm}$ and a linewidth of $\Delta\lambda=1\text{nm}$.
   c. A FP laser diode with a center wavelength of $\lambda=850\text{nm}$ and a linewidth of $\Delta\lambda=1\text{nm}$.
   d. A DFB laser diode with a center wavelength of $\lambda=1550\text{nm}$ and a linewidth of $\Delta f=20\text{MHz}$.

2. A communications link is designed with SMF28 optical fiber, a receiver that has a minimum detectable power is $P_{\text{min}}=-20\text{dBm}$, a laser that has a power of $10\text{mW}$, a center wavelength of $\lambda=1550\text{nm}$ and a linewidth of $\Delta\lambda=1\text{nm}$. The coupling between the laser and fiber is 60% and between the fiber and detector is 80%. The fiber comes in 5km sections. Each splice between fiber sections is 0.2dB and you need to design the link with a 3dB margin of error.
   a. What is the maximum link length?
   b. What is the maximum bit rate at this link length?
1. An optical communications link is constructed with SMF28 optical fiber (use $\lambda_0=1310\text{nm}$) and operated with a bit rate of $B=10\text{Gbps}$. What is the maximum distance using the following optical sources? (Ignore any attenuation limits.)
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(a) $\lambda=1310\text{nm}$, $B=10^9 \text{ bps}$

$\Delta\lambda = 30\text{nm}$

$\lambda > \lambda_c$ so single mode

From the SMF28 spec sheet $D(1310\text{nm}) = \frac{0.092}{4} \left[ \frac{1310 - 1310^2}{1310^3} \right] = 0$

So $\Delta T = \frac{1}{2} \left( \frac{\partial D}{\partial \lambda} \Delta\lambda^2 \right) L$

$\Delta T = \frac{1}{2} \left( 0.092 \right) \left( 30 \right)^2 L < \frac{1}{4B}$

$L < \frac{1}{(2)(0.092 \times 10^{-12})(30)^2(10^9)}$

$L < 0.61\text{km}$

(b) $\lambda=1310\text{nm}$

$\Delta\lambda = 1\text{nm}$

$B=10^9 \text{ bps}$

$D(1310) = 0$

$\Delta T = \left( \frac{1}{2} \right) \left( 0.092 \times 10^{-12} \right) \left( 1 \right)^2 L < \frac{1}{4B}$

$L < \frac{1}{(2)(0.092 \times 10^{-12})(1)(10^9)}$

$L < 5.3\text{km}$
(c) Since \( \lambda < \lambda_c \), the fiber becomes multimode

\[
V = \frac{2\pi}{\lambda} a \sqrt{n_i^2 - n_d^2}
\]

\[
= \frac{2\pi}{\lambda} a \sqrt{(n_i - n_d) \cdot n_d}
\]

\[
= \frac{2\pi}{\lambda} a \sqrt{\left(\frac{1}{n_i^2} - \frac{1}{n_d^2}\right) \cdot n_d^2}
\]

\[
= \frac{2\pi}{\lambda} a n_2 \sqrt{2a}
\]

\[
V = \left(\frac{2\pi}{0.85}\right) \left(4.1\right) \sqrt{(2)(0.0036)} \cdot n_2
\]

Look up \( n_2 \) from Table 1.1

\[
n_2 = 1.457 + \frac{(1.45 - 1.457)}{1 - 0.6528} (0.85 - 0.6328)
\]

\[
n_2 = 1.4529
\]

\[
V = 3.7363
\]

There are 2 modes with \( b_1 = 0.74 \) and \( b_2 = 0.379 \)

\[
D_{inter} = \frac{n_2 g}{c} \frac{(\Delta b_1 - b_2)}{2}
\]

\[
u s e: \quad n_g = 1.4677
\]

\[
D_{inter} = \left(\frac{1.4677}{3400}\right) \left(0.0036\right) \left(0.74 - 0.379\right)
\]

\[
= 6.376 \text{ ps}
\]

\[
= 6.376 \text{ ns}
\]

\[
\Delta T = DL = \frac{1}{48}
\]

\[
L = \frac{1}{(4)(6.376 \times 10^{-9})(10^6)}
\]

\[
L = 0.00392 \text{ km}
\]

\[
L < 3.92 \text{ m}
\]
(d) DFB laser

\[ \lambda = 1550 \text{nm} \]
\[ \Delta f_{\text{laser}} = 20 \text{ MHz} \]

\[ \Delta f_{\text{laser}} \ll \Delta f_{\text{mod}} \quad \text{so} \quad \Delta f = 2B \]

\[ f = \frac{c}{\lambda} \quad \frac{\Delta f}{\Delta \lambda} = -\frac{c}{\lambda^2} \]

\[ \Delta \lambda = \frac{\lambda^2}{c} \Delta f \]

\[ D_{\text{intra}} = \left( \frac{0.092}{4} \right) \left( 1550 - \frac{1310^4}{1550^3} \right) \]

\[ D_{\text{intra}} = 17.46 \text{ ps} \quad \frac{\text{km}}{\text{nm}} \]

\[ \Delta \lambda = \frac{\left( 1550 \times 10^{-9} \right)}{3 \times 10^8} \left( 2 \times 10^6 \right) = 0.16 \text{ nm} \]

\[ \Delta T = D \Delta \lambda = \frac{1}{4B} \]

\[ L = \frac{1}{(4)(10^6)(17.46 \times 10^{-12})(0.16)} \]

\[ L < 8.95 \text{ km} \]
2. A communications link is designed with SMF28 optical fiber, a receiver that has a minimum detectable power is $P_{\text{min}} = -20 \text{ dBm}$, a laser that has a power of 10mW, a center wavelength of $\lambda = 1550 \text{ nm}$ and a linewidth of $\Delta \lambda = 1 \text{ nm}$. The coupling between the laser and fiber is 60% and between the fiber and detector is 80%. The fiber comes in 5km sections. Each splice between fiber sections is 0.2dB and you need to design the link with a 3dB margin of error.
   a. What is the maximum link length?
   b. What is the maximum bit rate at this link length?

(a) $P_{\text{min}} = -20 \text{ dBm}$
   $P_t = 10 \text{ mW} = 10 \text{ dBm}$
   $\lambda = 1550 \text{ nm}$
   $\Delta \lambda = 1 \text{ nm}$
   coupling $0.6 = -2.2 \text{ dB}$
   $0.8 = -0.97 \text{ dB}$
   $\alpha = 0.19 \text{ dB/km}$

Power budget $10 + 20 - 2.2 - 0.97 - 0.19L = \frac{L}{5}(0.2) = 0$
$10 + 20 - 2.2 - 0.97 = L(0.19 + \frac{0.19}{5})$
$L \approx 116.6$

$N = \frac{L}{5} = 23.3$

$10 + 20 - 2.2 - 0.97 - 0.19L - (23)(0.2) = 0$
$\boxed{L = 117 \text{ km}}$

(b) From problem 1 $D_{\text{link}} = 17.46 \text{ ps/km-nm}$

$\Delta \Delta L = \frac{1}{48}$
$G = \frac{1}{40 \Delta \lambda}$

$B = \frac{1}{(4)(17.46 \times 10^{-9})^{117}} (1)
B = 1.22 \text{ Mb/s}$

Check $\Delta \Delta \lambda_{\text{mod}} = \frac{(1550 \times 10^{-9})^2}{3 \times 10^{-6}} (2)(122 \times 10^4) = 0.00195 \text{ nm} \ll 1 \text{ nm}$