Homework 4

1. An optical communications link is constructed with SMF28 optical fiber (use λ_o=1310nm) and operated with a bit rate of B=10Gbps. What is the maximum distance using the following optical sources? (Ignore any attenuation limits.)
    a. An infrared LED with a center wavelength of λ=1310nm and a linewidth of $\Delta\lambda$=30nm.
    b. A FP laser diode with a center wavelength of λ=1310nm and a linewidth of $\Delta\lambda$=1nm.
    c. A FP laser diode with a center wavelength of λ=850nm and a linewidth of $\Delta\lambda$=1nm.
    d. A DFB laser diode with a center wavelength of λ=1550nm and a linewidth of Δf=20MHz.

2. A communications link is designed with SMF28 optical fiber, a receiver that has a minimum detectable power is P_{min}=-20dBm, a laser that has a power of 10mW, a center wavelength of λ=1550nm and a linewidth of $\Delta\lambda$=1nm. The coupling between the laser and fiber is 60% and between the fiber and detector is 80%. The fiber comes in 5km sections. Each splice between fiber sections is 0.2dB and you need to design the link with a 3dB margin of error.
    a. What is the maximum link length?
    b. What is the maximum bit rate at this link length?

1. An optical communications link is constructed with SMF28 optical fiber (use λ_o=1310nm) and operated with a bit rate of B=10Gbps. What is the maximum distance using the following optical sources? (Ignore any attenuation limits.)
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(a) $\lambda = 1310nm$ $\quad B = 10^{10}$ bps
$\Delta\lambda = 30nm$
$\lambda > \lambda_c$ so single mode

From the SMF28 spec sheet $\quad D(1310nm) = \frac{0.092}{4}\left[1310 - \frac{1310^4}{1310^3}\right] = 0$

so $\Delta\tau = \frac{1}{2}\left(\frac{\partial D}{\partial\lambda}\Delta\lambda^2\right)L$

$$\Delta\tau = \frac{1}{2}(0.092)(30)^2 L < \frac{1}{4B}$$

$$L < \frac{1}{(2)(0.092\times10^{-12})(30)^2(10^{10})}$$

L < 0.6 Km

(b) $\lambda = 1310nm$
$\Delta\lambda = 1nm$
$B = 10^{10}$ bps
$D(1310) = 0$

$$\Delta\lambda_{mod} = \frac{\lambda^2}{c}\Delta f = \frac{(1310\times10^{-9})^2}{3\times10^8}(2)(10^{10})$$
$$= 0.114nm$$
$\Delta\lambda_{mod} \ll \Delta\lambda_{laser}$

$$\Delta\tau = \left(\frac{1}{2}\right)(0.092\times10^{-12})(1)^2 L < \frac{1}{4B}$$

$$L < \frac{1}{(2)(0.092\times10^{-12})(1)(10^{10})}$$

L < 543 Km

(c) Since $\lambda < \lambda_c$ the fiber become multimode

$$V = \frac{2\pi}{\lambda} a \sqrt{n_1^2 - n_2^2}$$

$$= \frac{2\pi}{\lambda} a \sqrt{(n_1 - n_2)\, 2n_2}$$

$$= \frac{2\pi}{\lambda} a \sqrt{\left(\frac{n_1 - n_2}{n_2}\right) 2n_2^2}$$

$$= \frac{2\pi}{\lambda} a\, n_2 \sqrt{2\Delta}$$

$$V = \left(\frac{2\pi}{0.85}\right)(4.1)\sqrt{(2)(0.0036)}\; n_2$$

Look up n_2 from Table 1.1

$$n_2 = 1.457 + \frac{(1.45 - 1.457)}{1 - 0.6328}(0.85 - 0.6328)$$

$$n_2 = 1.4529$$

$$V = 3.7363$$

There are 2 modes with $b_1 = 0.741$ and $b_2 = 0.379$

$$D_{inter} = \frac{n_{2g}}{c}(\Delta)(b_1 - b_2)$$

use $n_g = 1.4677$

$$D_{inter} = \left(\frac{1.4677}{3\times10^8}\right)(0.0036)(0.741 - 0.379)$$

$$= 6.376 \frac{ps}{m}$$

$$= 6.376 \frac{ns}{km}$$

$$\Delta T = DL = \frac{1}{4B}$$

$$L = \frac{1}{(4)(6.376\times10^{-9})(10^{10})}$$

$$L = 0.00392 \text{ km}$$

$L < 3.92\text{m}$

(d) DFB laser

$\lambda = 1550\,nm$

$\Delta f_{laser} = 20\ MHz$

$\Delta f_{laser} << \Delta f_{mod}$ so $\Delta f = 2B$

$f = \frac{c}{\lambda}$ $\quad \frac{\Delta f}{\Delta \lambda} = -\frac{c}{\lambda^2}$

$\Delta \lambda = \frac{\lambda^2}{c} \Delta f$

$D_{intra} = \left(\frac{0.092}{4}\right)\left(1550 - \frac{1310^4}{1550^3}\right)$

$D_{intra} = 17.46 \frac{ps}{km\ nm}$

$\Delta \lambda = \frac{(1550 \times 10^{-9})}{3 \times 10^8} (2)(10^{10}) = 0.16\ nm$

$\Delta T = D \Delta \lambda L = \frac{1}{4B}$

$L = \frac{1}{(4)(10^{10})(17.46 \times 10^{-12})(0.16)}$

$L < 8.95\ km$

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   a. What is the maximum link length?
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(a) $P_{min} = -20$ dBm
$P_t = 10\text{ mW} = 10$ dBm
$\lambda = 1550$nm
$\Delta\lambda = 1$nm
coupling $0.6 = -2.2$ dB
$0.8 = -0.97$ dB
$\alpha = 0.19$ dB/km

Power budget $10+20-2.2-0.97-0.19L-\left(\frac{L}{5}\right)(0.2) = 0$

$10+20-2.2-0.97 = L\left(0.19+\frac{0.2}{5}\right)$

$L \approx 116.6$

$N = \frac{L}{5} = 23.3$

$10+20-2.2-0.97-0.19L-(23)(0.2) = 0$

L = 117 Km

(b) from problem 1 $D_{intra} = 17.46 \frac{ps}{km \cdot nm}$

$\Delta\lambda DL = \frac{1}{4B}$

$B = \frac{1}{4DL\Delta\lambda}$

$B = \frac{1}{(4)(17.46\times10^{-12})(117)(1)} =$

B = 122 Mb/s

check $\Delta\lambda_{mod} = \frac{(1550\times10^{-9})^2}{3\times10^8}(2)(122\times10^6) = 0.00195\text{nm} \ll 1\text{nm}$