Homework #2

For all problems in this homework assignment assume a wavelength of λ=1550nm.

1. A symmetric slab waveguide has a slab index of refraction of n_2=1.5 and surrounding refractive indices of n_1=1.49. What is the range of waveguide thicknesses for which this waveguide will be single mode?
2. A slab waveguide has a thickness of d=5μm, a slab index of refraction of n_2=1.5 and surrounding refractive indices of n_1=1.49.
    a. How many TE modes does the waveguide support?
    b. How many TM modes does the waveguide support?

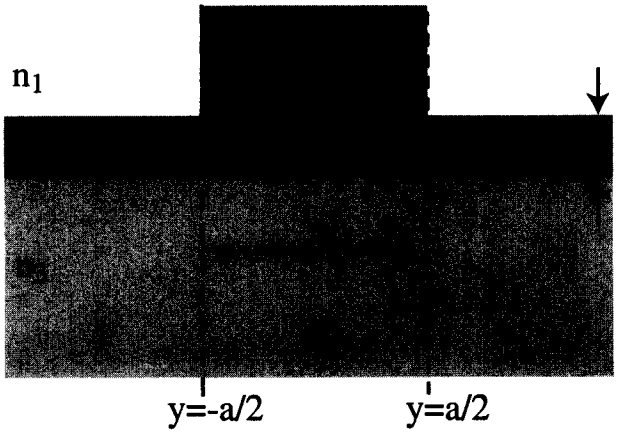


Figure 1

3. Figure 1 shows a ridge waveguide. $n_1=n_3=1.5$ and $n_2=1.6$.
    a. Calculate the ridge thickness t such that the second mode is cut-off at the wavelength of λ=1μm. (The single mode operating range is $\lambda>1\mu m$)
    b. If $d=0.8t$ and $a=10\mu m$, calculate the effective indices of all of the supported modes.
4. Calculate the mode designations for the lowest 5 modes of a step index optical fiber and calculate the normalized frequency of their cut-offs.

(1) $n_1 = 1.49$
$n_2 = 1.50$

Single mode requires

$$\frac{\pi}{\lambda} d \sqrt{n_2^2 - n_1^2} < \frac{\pi}{2}$$

$$d < \frac{\lambda}{2\sqrt{1.5^2 - 1.49^2}}$$

$$\boxed{d < 4.48\,\mu m}$$

(2) $d = 5\,\mu m$
$n_1 = 1.5$
$n_2 = 1.49$

(a) The equations for TE modes are

$$\frac{hd}{2}\tan\left(\frac{hd}{2}\right) = \frac{qd}{2} \quad \text{or} \quad -\frac{hd}{2}\cot\frac{hd}{2} = \frac{qd}{2}$$

and

$$\left(\frac{hd}{2}\right)^2 + \left(\frac{qd}{2}\right)^2 = \left[(n_2 k_0)^2 - (n_1 k_0)^2\right]\left(\frac{d}{2}\right)^2$$

$$= \left(\frac{\pi}{\lambda} d\right)^2 \left(n_2^2 - n_1^2\right)^2 \equiv V^2$$

Need to find the zeros of the top equations that are smaller than V

$$V = \pi \frac{(5)}{1.55}\sqrt{1.5^2 - 1.49^2}$$

$$= 0.56\pi$$

Let $\frac{hd}{2} = x$ and $\frac{qd}{2} = y$ $\quad x\tan x = y$ and $-x\cot x = y$

$x\tan x = y$: zeros at $0, \pi, 2\pi$ $\quad$ $-x\cot x = y$: zeros at $\pi/2, 3\pi/2 \ldots$

so we have TE_0 and TE_1 modes

(b) for TM modes we use

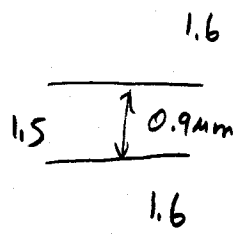
$$x\tan x = \frac{n_2^2}{n_1^2} y \quad \text{and} \quad -x\cot x = \frac{n_2^2}{n_1^2} y$$

These have the same zeros so

$\boxed{TM_0 \text{ and } TM_1 \text{ modes}}$

(3) $n_1 = n_3 = 1.5$
$n_2 = 1.6$

(a) $\lambda = 1\mu m$ find t
we want $V = \frac{\pi}{2}$

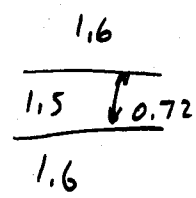


$$\left(\frac{\pi}{1}\right)(t)\sqrt{1.6^2 - 1.5^2} = \frac{\pi}{2}$$

$$t = 0.90\ \mu m$$

at $\lambda = 1\mu m$, $t = 0.9\ \mu m$ $\quad n_{eff} = 1.5655$

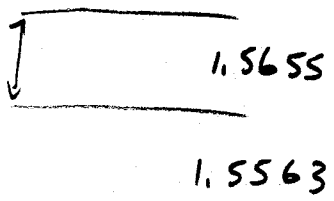
(b) with $n_1 = n_3 = 1.5$
$n_2 = 1.6$
$d = 0.7\mu m$ $\quad n_{eff} = 1.5563$



Now the effective index method

4 modes

$N_{eff} = 1.5649,\ 1.5633,\ 1.5606,\ 1.5573$



(4) Calculate the mode designations for the lowest 5 modes

The mode equations are $\quad ha\ \frac{J_{\ell+1}(ha)}{J_\ell(ha)} = qa\ \frac{K_{\ell+1}(qa)}{K_\ell(qa)}$

at cut-off $qa = 0$
so we need $\lim_{qa \to 0} qa\ \frac{K_{\ell+1}(qa)}{K_\ell(qa)}$

$$\lim_{qa \to 0} qa\ \frac{K_1(qa)}{K_0(qa)} = 0$$

$$\lim_{qa \to 0} qa\ \frac{K_2(qa)}{K_1(qa)} = 2$$

$$\lim_{qa \to 0} qa\ \frac{K_3(qa)}{K_2(qa)} = 4$$

For the LP_{0m} modes

$$\frac{x J_1(x)}{J_0(x)} = 0$$

$$x J_1(x) = 0 \quad \text{at} \quad x = 0,\ 3.8318,\ 7.0157$$

For the LP_{1m} modes

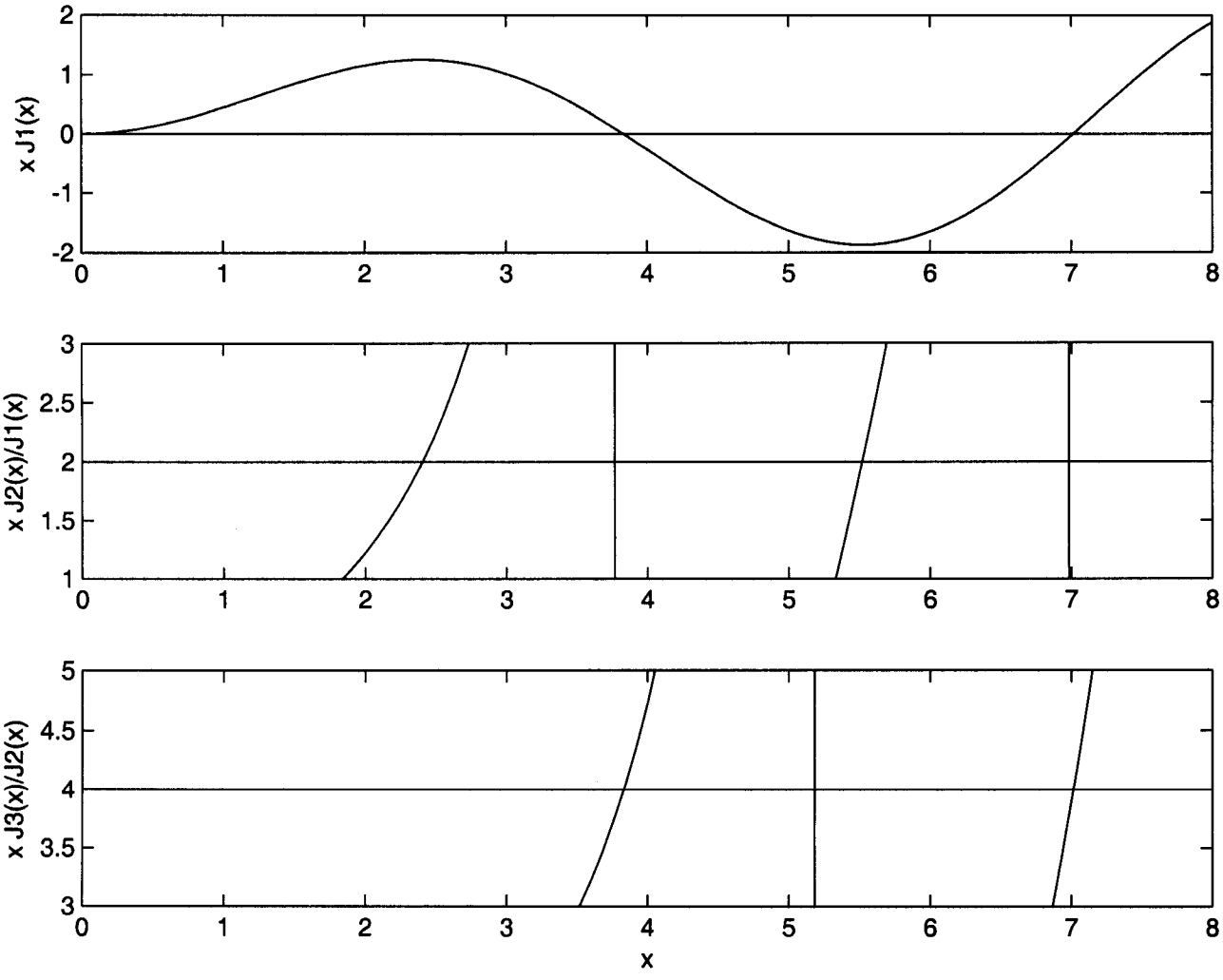
$$\frac{x J_2(x)}{J_1(x)} = 2 \quad \text{at} \quad x = 2.405,\ 5.5201$$

For LP_{2m} modes

$$\frac{x J_3(x)}{J_1(x)} = 4 \quad \text{at} \quad x = 3.8318,\ 7.0151$$

Modes are

LP_{01}	$V < 2.405$
LP_{11}	$V < 3.8318$
LP_{02} and LP_{21}	$V < 5.5201$
LP_{12}	$V < 7.0151$


x J1(x)
x J2(x)/J1(x)
x J3(x)/J2(x)
x