Experimental results: The Addie stochastic learning automaton was tested using a storage oscilloscope to observe the state trajectories directly. Learning curves obtained with the $L_{R-P}$ scheme are shown in Fig. 2a. The curves show how the degree of expediency increases with reward/punishment ratio $\gamma$ ($\gamma = (1 - \beta)/(1 - \delta)$). It should be noted that the counter implementation allows a proper initial condition to be set up, corresponding to random state selection at time $t_0$, i.e. $p_1(0) = 0.5$. For these experiments, the Addie clock was set at 10 MHz and the noise generator clock at 1 MHz, while the main system clock consisted of 100 ns pulses with 10 kHz repetition rate.

The results obtained with the optimal $L_{R-I}$ scheme are illustrated in Fig. 2b. With this scheme, the automaton exhibits virtually full convergence to $p_1 = 1$ or $p_2 = 1$. It is known that the $L_{R-R}$ scheme is comparable in expediency to the $L_{R-I}$ scheme when $\gamma$ is high but the $L_{R-R}$ scheme is expected to exhibit a slower rate of convergence. The results obtained with a $L_{R-R}$ scheme are shown in Fig. 2c. As expected the degree of expediency increases with $\gamma$.

The general conclusion to be gained from the above results on linear schemes is that there is little difference in expediency between the schemes with high $\gamma$-factor. The criterion of relative convergence rates, which is stressed in the reported software simulation studies, is less important in the work reported here. Indeed, there appears to be no discernible difference in learning time between the schemes investigated. The chief determining factor in the transient response of the hardware systems is the output Addie. This element has a restricted bandwidth in order to obtain an acceptable compromise between convergence speed and steady-state variance.

Conclusions: The above results have verified that the Addie stochastic learning automaton has very satisfactory learning characteristics. Although its operation involves a certain amount of serial processing, similar learning times to the flip-flop automaton have been obtained. Indeed the results obtained previously for the $L_{R-P}$ scheme have been virtually duplicated by the new circuit, which has of course the added advantage of being able to implement a more comprehensive range of reinforcement schemes. The ability to optimise in less than 10 ms represents a significant development, particularly in view of the proposed extension to systems of much larger state order, embodying the same basic Addie stochastic automata within a hierarchical structure.

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MATERIAL DISPERSION IN LIGHTGUIDE GLASSES

Indexing terms: Optical fibres, Glass

Material dispersion measurements are reported on six characteristic lightguide glass compositions. The measurements were made on bulk specimens and cover the wavelength range from 0.8 to 1.5 μm. It is observed that in these silicate glasses the wavelength at which material dispersion is zero is in all cases greater than 1.2 μm.

Introduction: In order to increase the bandwidth of lightguides it is desirable to minimise material dispersion—the wavelength dependence of the light group velocity in the transmission medium. Measurements of material dispersion have been reported on certain bulk samples of lightguide glasses1,2 and also on optical fibre specimens.3-5 Both types of analyses are being pursued at this laboratory. Although bulk specimens may not duplicate the materials comprising those in lightguides, dispersion measurements on bulk specimens provide the following advantages: (a) accurate sample chemical composition determinations can be made on bulk material; (b) measurements are free of waveguide effects which may complicate dispersion analyses on fibres; (c) measurements can be performed on materials contemplated for optical waveguide cladding such as B2O3-SiO2 compositions; (d) dispersion can be analysed in materials prior to the achievement of low-loss lightguides made from them.

The results of such measurements obtained on characteristic compositions of materials currently used in waveguides are reported herein.

Experimental: The glass compositions on which material dispersion is reported are listed in Table 1. Glass A-D were prepared at this laboratory by r.f. plasma fusion of vapour or preheated powder, glass E was prepared by Hereaus Quarzschmelze and glass F was prepared at this laboratory by a conventional high-purity crucible melting technique. The compositions reported are those indicated by chemical analysis of the prepared glass and are accurate to ±0.1 mole %.

Reflective indices were measured at the wavelengths given in Table 2. The measurement method utilised was the

Table 1 COMPOSITIONS AND ZERO MATERIAL DISPERSION FOR GLASSES STUDIED

<table>
<thead>
<tr>
<th>Sample</th>
<th>Composition</th>
<th>Zero material dispersion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>µm</td>
</tr>
<tr>
<td>A</td>
<td>Quenched SiO2</td>
<td>1.284</td>
</tr>
<tr>
<td>B</td>
<td>13.5GeO2 : 86.5SiO2</td>
<td>1.274</td>
</tr>
<tr>
<td>C</td>
<td>9.1P2O5 : 90.9SiO2</td>
<td>1.284</td>
</tr>
<tr>
<td>D</td>
<td>13.5B2O3 : 86.5SiO2</td>
<td>1.231</td>
</tr>
<tr>
<td>E</td>
<td>1.0F : 99.0SiO2</td>
<td>1.284</td>
</tr>
<tr>
<td>F</td>
<td>16.9Na2O : 32.5B2O3 : 50.6SiO2</td>
<td>1.283</td>
</tr>
</tbody>
</table>

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minimum deviation technique. The method and apparatus are essentially described in Reference 1. The measured wavelength range was extended in order to avoid the extrapolation previously necessary to obtain information about dispersion in the 1-0 to 1-5 μm wavelength range in which material dispersion approaches zero for many oxide glasses of interest.

Table 2 WAVELENGTHS AT WHICH REFRACTIVE INDEX WAS MEASURED

<table>
<thead>
<tr>
<th>λ (μm)</th>
<th>n</th>
<th>d^2n</th>
<th>c d^2n^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-435835</td>
<td>0-807902</td>
<td>0-004439</td>
<td>0-000039</td>
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<tr>
<td>0-479991</td>
<td>0-852111</td>
<td>0-005239</td>
<td>0-000049</td>
</tr>
<tr>
<td>0-508582</td>
<td>0-917224</td>
<td>0-006039</td>
<td>0-000059</td>
</tr>
<tr>
<td>0-546073</td>
<td>1-002439</td>
<td>0-006839</td>
<td>0-000069</td>
</tr>
<tr>
<td>0-589262</td>
<td>1-08303</td>
<td>0-007639</td>
<td>0-000079</td>
</tr>
<tr>
<td>0-643847</td>
<td>1-367351</td>
<td>0-008439</td>
<td>0-000089</td>
</tr>
<tr>
<td>0-734620</td>
<td>1-52958</td>
<td>0-009239</td>
<td>0-000099</td>
</tr>
</tbody>
</table>

A state-of-the-art infrared detector which exhibits high sensitivity through 1-7 μm was used for the measurements above 1-083 μm and a silicon diode detector was used for all other wavelengths measured. All measurements were made at 23±5°C. The technique requires prismatic specimens with accurately determined prism angles and optically polished prism faces. Prisms were made from the above glasses by conventional optical polishing techniques. They have prism angles of 30–60° and optically polished faces with finishes better than 1/10 wave. The prism faces have areas of about 4 cm².

The bulk SiO₂ glass used in this examination was high purity low-water-content plasma fused silica made by Nassau and Shiever. The reason this data was chosen over extensive National Bureau of Standards measurements is that the NBS measurements were made on annealed samples and it is known that SiO₂ undergoes an anomalous change in refractive index as a result of quenching. Details on the extent of this effect were reported by Fleming and Shiever. Fibre drawing causes glasses to be in a quenched state. Because of the nature of the preparation technique, the plasma fused SiO₂ of Nassau and Shiever is also quenched as is the case for glasses prepared by r.f. plasma fusion of presintered powder. The thermal history of these samples may not be identical to that of fibre specimens but is more representative of theirs than that of annealed samples.

The Na₂O·B₂O₃·SiO₂ glass used was obtained from a piece of cane drawn from the fluid melt. This may aid in producing a glass more closely representing the properties of a fibre specimen.

The resultant refractive-index values for each material were fitted to a three term Sellmeier dispersion relation of the form:

\[ n^2 - 1 = \sum_{i=1}^{3} \frac{A_i \lambda_i^2}{\lambda^2 - l_i^2} \]  

where

- \( n \) is the refractive index
- \( \lambda \) is the wavelength
- \( A_i \) are constants related to material oscillator strengths
- \( l_i \) are oscillator wavelengths.

Table 3 FITTED SELLMEIER COEFFICIENTS FOR LIGHTGUIDE GLASSES STUDIED

<table>
<thead>
<tr>
<th>Sample</th>
<th>( A_1 )</th>
<th>( l_1 )</th>
<th>( A_2 )</th>
<th>( l_2 )</th>
<th>( A_3 )</th>
<th>( l_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0-696750</td>
<td>0-069066</td>
<td>0-408218</td>
<td>0-115662</td>
<td>0-890815</td>
<td>9-900559</td>
</tr>
<tr>
<td>B</td>
<td>0-711040</td>
<td>0-064270</td>
<td>0-451885</td>
<td>0-129408</td>
<td>0-704048</td>
<td>9-425478</td>
</tr>
<tr>
<td>C</td>
<td>0-695790</td>
<td>0-061568</td>
<td>0-452497</td>
<td>0-119921</td>
<td>0-712513</td>
<td>8-656641</td>
</tr>
<tr>
<td>D</td>
<td>0-690618</td>
<td>0-061900</td>
<td>0-401996</td>
<td>0-123662</td>
<td>0-898817</td>
<td>9-098960</td>
</tr>
<tr>
<td>E</td>
<td>0-691116</td>
<td>0-068227</td>
<td>0-399166</td>
<td>0-116460</td>
<td>0-890423</td>
<td>9-993707</td>
</tr>
<tr>
<td>F</td>
<td>0-796468</td>
<td>0-094359</td>
<td>0-497614</td>
<td>0-093386</td>
<td>0-358924</td>
<td>5-999652</td>
</tr>
</tbody>
</table>

The fit was performed by a nonlinear least-square fitting routine.

Measured refractive indices are accurate to better than ±0-00005. The average deviation in the fitted index values was better than ±0-00002 with fitted values exhibiting precision better than the measured accuracy at each point. The parameters of the fitted three-term Sellmeier relationship are listed in Table 3 for each material.

The material dispersion given by the quantity

\[ \lambda \frac{d^2n}{d\lambda^2} \]

has been determined for the six materials listed in Table 1. For the calculation, \( d^2n/d\lambda^2 \) was obtained exactly from the second derivative of eqn. 1. The resultant material dispersion against wavelength for the samples is indicated in Fig. 1. The material dispersion of the fluorine doped samples was for all practical purposes the same as that for pure SiO₂ glass and, therefore, does not appear independently in Fig. 1. Of interest is the wavelength \( \lambda_0 \) at which expr. 2 goes to zero for lightguide glasses. These values are listed in Table 1.

Fig. 1. Dependence of material dispersion on wavelength for lightguide glasses

- Na₂O·B₂O₃·SiO₂
- P₂O₅·SiO₂
- B₂O₃·SiO₂
- GeO₂·SiO₂
- SiO₂

Material dispersion measurements in glass composition B and a composition similar to D were reported in Reference 1. The thermal histories of these two samples differ from those of the previously analysed specimens which were respectively similar in composition. This variation is slight for B but is greater for D because of the change in sample preparation technique. The thermal history is a parameter which influences refractive index dispersion in glasses. Quenched and annealed samples of the same composition exhibit different refractive indices. Thus refractive indices for sample B differ slightly from those measured before (<0-0001) and those for sample D differ more (~0-003). The fit of relation 1 to the
Discussion: Comparing the $B_2O_3$-$SiO_2$ data with values measured on lightguides comprised of a $B_2O_3$-$SiO_2$ cladding (14 mole % $B_2O_3$) and a fused silica core, it is observed that the bulk sample exhibits lower material dispersion than the fibre specimen. It should be noted that the fibre has a graded refractive-index profile with maximum $B_2O_3$ content in the centre. It is expected therefore that the dispersion measured in the fibre is a weighted average of the dispersive behaviours of all compositions present in the guiding portion of the fibre. This being considered, the comparison remains qualitatively consistent since the fibre measurement is lower than expected for pure fused silica ($\lambda_0 = 1.262$ compared to 1.284 $\mu m$).

$Na_2B_2O_3$-$SiO_2$ glass does not exhibit material dispersion significantly higher than does fused silica. The results support an earlier finding that the addition of $B_2O_3$ to a silicate glass system moves the point at which material dispersion goes to zero to a shorter wavelength.

The material dispersion behaviour for the $P_2O_5$-$SiO_2$ glass reported here agrees well with observations obtained by Payne et al. ($\lambda_0 \sim 1.272 \mu m$) on step index fibre specimens and also with observations made by Cohen and Lin ($\lambda_0 \sim 1.283 \mu m$) on graded index fibres, all specimens having core compositions similar to the composition of the bulk specimen. As observed, the addition of $P_2O_5$ to $SiO_2$ does not significantly alter the material dispersion exhibited by fused silica. Therefore the change in core composition of a graded index fibre is not expected to cause changes in the apparent material dispersion as measured by the pulse delay technique.

In contrast, addition of $GeO_2$ to silica increases the material dispersion and shifts the point of zero material dispersion to longer wavelengths. Graded index $GeO_2$-$SiO_2$ core fibres exhibit higher material dispersion than silica, $P_2O_5$-$SiO_2$ core fibres. However, the magnitude of the increase in a graded fibre is not as great as that measured in the bulk samples having $GeO_2$ content similar to the maximum present in the fibre core ($\lambda_0 \sim 1.33$ as compared to 1.38 $\mu m$).

It should be noted that comparison with this particular fibre measurement is complicated by the fact that the fibre core contained 4 mole % $B_2O_3$ which, as indicated, lowers the material dispersion. Payne and Hartog have determined material dispersion in a step index fibre with a core reported to be 131 $GeO_2$: 86.9 $SiO_2$. They found $\lambda_0$ occurred at 1.355 $\mu m$. Although higher than the graded index fibre $\lambda_0$ is still below that measured on a bulk specimen. This discrepancy may result from waveguide effects in the step index fibre.

Material dispersion has been measured in a number of silicate based glasses in addition to those reported here. It is found that the wavelength at which material dispersion is zero is in all cases greater than 1 $\mu m$. In concurrence with results obtained by other techniques there is no evidence that a suitable dopant exists which, when added to a silicate glass, can shift the point of zero material dispersion to wavelengths in the 0.8 to 1 $\mu m$ range where sources are currently available.

Acknowledgment: The author thanks N. Treber and R. Rau of Heraeus for providing the $F$ doped $SiO_2$ sample, J. W. Shiever and K. Nassau for the fused silica sample, and A. D. Pearson and W. R. Northover for the $Na_2O$-$B_2O_3$-$SiO_2$ sample. He also appreciates helpful discussions with L. G. Cohen and Chinlin Lin.

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5 also Appl. Opt., 1977, 16, pp. 3136-3139, and unpublished work

NEW CONTACT RESISTANCE PROFILING METHOD FOR THE ASSESSMENT OF III-V ALLOY MULTILAYER STRUCTURES

Indexing terms: Contact resistance, Doping profiles, III-V semiconductors

A practical technique for the assessment of multilayer III-V structures is described which is based on the measurement of the contact resistance between a tungsten carbide probe and an angle lapped sample surface. The technique has been applied to homostucture and double heterostructure layers of GaAs, GaAlAs, InP and GaAsP, and quantitative information on the doping profiles has been obtained by calibration against binary material samples of known impurity levels.

Introduction: The use of multilayer epitaxial structures in III-V alloys such as GaAs$_x$AlAs$_{1-x}$, Ga$_{1-x}$In$_x$As and Ga$_{1-x}$In$_x$As$_{1-y}$P$_y$ has led to great advances in several opto-electronic devices such as I.E.D.s, lasers, solar cells and photodetectors. The optimisation of these structures can present problems because the device requirements demand very thin layers, high impurity concentrations, and alloy compositions which may be nonuniform and vary from layer to layer. Furthermore device performance may depend on the exact nature of the layer interfaces which can be graded in composition and doping, due to interdiffusion and mixing. In the absence of any suitable detailed assessment techniques one must trust that the layers grown have similar properties to those of single assessment layers grown under conditions under near identical conditions, on high resistivity substrates. Such layers can be assessed by Hall and compositional measurements. It is more desirable, however, that each complete structure be assessed after growth.

Several profiling methods are used routinely for semiconductor layer assessment. The Schottky barrier profiler can be used to assess the doping level in the depleted material below a reverse biased metal/semiconductor contact. However, in highly doped material, electrical breakdown of the Schottky barrier occurs for quite thin depletion widths and hence a sequence of...